PHYSICAL LABORATORY EXPERIMENTS

MECHANICS, OPTICS and HEAT

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PART I—MECHANICS.

CONTENTS.

PAGE
- Units of Length and Estimation of Tenths . . . 3
- Verniers . . . 7
- Area Measurements . . . 10
- Measurement of Thickness . . . 16
  - The Micrometer Caliper . . . 16
  - The Spherometer . . . 17
  - The Optical Micrometer . . . 19
- The Spirit Level . . . 21
- Measurement of Mass. (General Discussion) . . . 27
  - Theory of the Balance . . . 29
  - The Use of a Balance . . . 33
  - Methods of Weighing . . . 34
  - Double Weighing . . . 36
  - Reduction of Weighings to Vacuo . . . 37
- The Balance, Part I. (Methods of Weighing) . . . 40
- The Balance, Part II. (Sensitiveness and Ratio of Arms) . . . 42
- The Balance, Part III. (Calibration of Set of Weights) . . . 45
- Specific Gravity. (General Discussion) . . . 47
- Specific Gravity of Solids, I. (Specific Gravity-bottle) . . . 49
  - Discussion of Corrections . . . 52
- Specific Gravity of Solids, II. (Archimedes' Principle) . . . 55
- Specific Gravity of Liquids, I. (Specific Gravity-bottle) . . . 56
- Specific Gravity of Liquids, II. . . . 58
  - Sprengel-Ostwald Picnometer . . . 58
  - Mohr-Westphal Balance . . . 60
- The Jolly Balance . . . 62
- Barometric Measurements. (General Discussion) . . . 67
  - Types of Barometer . . . 71
- Comparison of Barometers . . . 79
- Meteorological Instruments . . . 81
- Boyle's Law, I. . . . 86
- Boyle's Law, II. . . . 90
- Law of the Pendulum . . . 94
- The Method of Coincidences. (General Discussion) . . . 97
- The Physical Pendulum . . . 99
- The Metronome Pendulum . . . 104
- Law of Freely Falling Bodies . . . 108
  - The Tuning-fork Chronograph . . . 108
- Modulus of Elasticity . . . 115
- Appendix . . . i–ix

13169

UNITS OF LENGTH AND ESTIMATION OF TENTHS.

Object.—The object of this experiment is twofold—first, to determine the value of the inch in terms of the millimeter, and second, to give the student practice in estimating by eye, tenths of a small division. The ability to make accurately such estimations is of great importance in all scientific work as the experimenter is continually obliged to estimate fractions, usually tenths, of the smallest division of his instrument e.g. tenths of a degree, tenths of a millimeter, etc. The computation also introduces the student to some fundamental matters in the precision discussion of direct measurements.

Apparatus.—The apparatus consists of a micrometer screw, *M*, mounted horizontally and carrying a movable nut on which is a fine reference mark, *N*. The nut moves between two horizontal scales, S_1 and S_2, the graduations of which are flush with the reference mark on the nut, thus eliminating errors in reading arising from parallax. The scale S_1 is graduated in twentieths of an inch. The scale S_2 is graduated in millimeters corresponding to the pitch of the micrometer screw. One complete revolution of the screw moves N through exactly one division of S_2, and this scale may, therefore, be used for recording the total number

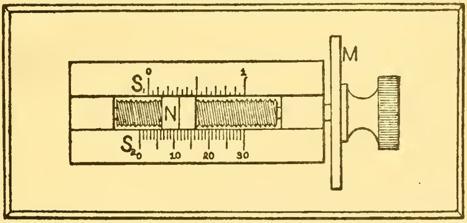


Fig. 1.

of revolutions made by the screw. Fractions of a revolution are read off on the graduated head of the micrometer (divided into 100 parts), with respect to a fixed reference mark not shown in the figure. The instrument is so adjusted that when the zero of the micrometer head is set on the reference mark, N coincides with the divisions on S_2.

Procedure.—*First.*—To measure the inch in terms of the millimeter.

Set the reference mark N exactly on the zero division of the inch scale and record the position of N on the millimeter scale to thousandths of a millimeter by estimating the position of the micrometer head with respect to the reference mark at the base of the instrument. For example, if N stands between divisions 2 and 3 on S_2 and the reference division for reading M is half way between the 43 and 44 graduations of the micrometer head, the reading will be 2.435 mm. In setting N *always* bring it up to the mark from the same side, e.g., turning a right-handed screw clock-wise. This is necessary to eliminate *backlash,* i.e., the play of the screw in the nut, which is likely to be present, sometimes to a large extent, in instruments involving the use of the screw. It is evident that if all settings be made from the same direction this source of error will be eliminated in taking the difference of two consecutive series of settings. This point should be borne in mind in all subsequent work. Make nine independent settings of N on both the first and last division of the inch, recording the position of N each time, as described above. The difference of the means of these two series of observations will evidently be the best representative value of one inch expressed in millimeters.

Computation.—Find the mean, the average deviation of a single setting and the average deviation of the mean setting for each series. From these, compute the value of the inch in millimeters and the average deviation and percentage deviation of this result (see precision discussion).

Second.—Estimation of tenths of a small division (one-twentieth of an inch).

Compute (in the laboratory) from the value of the inch in terms of the pitch of the micrometer screw, what the setting of the micrometer should be when N is set exactly at 0.1, 0.2, 0.3, etc., . . . of the distance between two adjacent graduations of the inch scale.

Thus, suppose we take the first small division of S_1. If $a =$ value of 1 inch in millimeters then $\frac{a}{20} =$ value of one small division of S_1, and $\frac{1}{10} \times \frac{a}{20}$ added to the *mean* value of the series of settings of N on the first graduation of S_1 will give the most probable value of the micrometer reading when N stands at $\frac{1}{10}$ the distance between this division and the next. In this way compute the settings for 0.2, 0.3 . . . 0.9 of this small division.

Having computed the value of these settings *set* the micrometer at the computed values, and observe carefully the position of N on the inch scale. In other words, let the eye see how the various tenths of the division really appear. It is well to train the eye in the following way: Set the micrometer, say at 0.3 of the division, and after observing the position of N carefully, move it to one side, and then set it by the eye as nearly as possible to the original position without looking at the micrometer. Then compare the reading of the micrometer with the computed value. The difference gives a measure of the precision with which the observer is capable of estimating that particular tenth. Since the distance estimated in each case is $\frac{1}{10}$ of $\frac{1}{20}''$ or $\frac{1}{200}'' = 0.005''$ the observer must be able to estimate to less than one-half this distance if his estimation is to be correct to the *nearest* tenth; that is, the estimated settings must differ from the true settings by less than 0.0025″ or about 0.063 mm. Students should practice until they can readily estimate tenths to approximately 0.05 mm. When some skill has been acquired record a series of independent estimations from 0.1 to 0.9. A comparison with the computed values should not be made until the estimated series of observations is complete. In order that this may represent fairly the observer's ability to estimate tenths no observation should be rejected if found to differ too widely from the computed values.

Computation.—Compute the average deviation of a single eye estimation from the differences between the estimated and computed values. This will give a measure of the precision with which the observer can estimate tenths.

Precision Discussion.—*Do the measurements on the length of the inch scale in terms of the micrometer screw indicate that the experimental errors are greater or less than the errors inherent in the instrument, e.g., errors due to faulty graduation of the scales or cutting of the screw?*

To answer this question, we must compare the *deviation* measure of the final result with the amount by which the result differs from the true value (if this be known) or from that obtained by the use of some other instrument.

If m_1 and m_2 are the *mean* settings on the inch scale and $\delta_1 = \pm (A.D.)_1$ and $\delta_2 = \pm (A.D.)_2$, their respective deviation measures, then the deviation in $M = m_1 - m_2$ due to observational errors alone may be as small as $\delta_1 - \delta_2$ if the deviations happen to be of such sign as to tend to neutralize each other, or as large as $\delta_1 + \delta_2$ if they both happen to effect the result with the same sign. Since their sign is not known, and they are equally likely to be plus or minus, it can be shown that the most probable uncertainty in M is obtained by squaring the deviation which each measurement individually produces in the final result, adding these together, and taking the square root of their sum, *i.e.*, in the above case

$$\Delta = \pm\sqrt{\delta^2{}_1 + \delta_2}$$

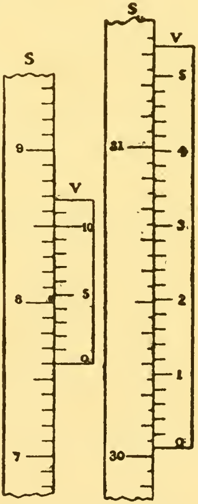
will be the probable deviation in M. If this value is appreciably less than the deviation between the observed and true value of the inch ($1'' = 25.4005$ mm.), it may be concluded that the apparatus contains some instrumental error or is used under conditions which are not standard. If, on the other hand, the difference between the true and computed value is of the same order of magnitude or is less than the deviation in the final result, it may be concluded that, within the limit of observational error, constant errors in the apparatus are not excessive or may be neglected.

Questions.—1. Do you think there is a source of constant error in the method or apparatus used? Why?

VERNIERS.

Object.—The object of this experiment is to familiarize the student with the theory and reading of various types of verniers.

Discussion.—The vernier, so called from its inventor Pierre Vernier, is a device for reading with accuracy and facility a fraction of a scale division, thereby avoiding the necessity of minute and expensive graduations or of eye estimation. It is universally used on barometers, cathetometers, divided circles, and other instruments of precision. The principle of the device is as follows: moving flush and parallel with the scale proper *S*, which is usually fixed, is a second scale *V*, the vernier, graduated into a number of equidistant parts *n*, of such length that the total number of parts *n* of the vernier is exactly equal in length to $n \pm 1$ smallest divisions of the scale. Thus in *A*, Fig. 2, ten divisions of the vernier equal nine of the scale; in *B*, twenty-five of the vernier equal twenty-four of the scale. The actual length of a vernier division is, therefore, $\frac{1}{n}$ less or greater than the length of the smallest division of the scale, as shown by the following demonstration.



A. 7.60 B. 30.026

Fig. 2.

Demonstration.—If *n* vernier divisions equal $n \pm 1$ scale divisions, and *s* and *v* are the lengths of a scale and vernier division respectively, then

$$nv = (n \pm 1)\, s,$$

$$v = \frac{(n \pm 1)}{n}\, s,$$

$$\therefore s - v = \frac{1}{n}\, s,$$

or

$$v - s = \frac{1}{n}\, s.$$

This difference, $s - v$ or $v - s$, between the length of a scale and vernier division, is called the *least count* of the vernier, and is always found by dividing the value of the smallest scale division by the total number of divisions into which the vernier is divided. Thus in A the smallest scale division is seen to be $\frac{1}{10}$ of unity or 0.1, and the vernier is divided into 10 parts; hence the vernier will read to (or the least count will be) $\frac{1}{10}$ of 0.1 or 0.01; in B, the smallest scale division is $\frac{1}{20}$ of unity or 0.05, and the vernier is divided into 25 parts; hence it reads to (or its least count is) $\frac{1}{25}$ of 0.05 or 0.002; again, if the smallest scale division is $\frac{1}{3}$ of a degree or $20'$ and the vernier is divided into 40 parts, it will read to $\frac{1}{40}$ of $20'$ or $\frac{1}{2}' = 30''$.

Consider now vernier A in figure 2. In moving the zero division of the vernier from a coincidence with the scale division 7.6, to a coincidence with the next adjacent division 7.7, it is evident that there will be ten intermediate coincidences of the successive vernier divisions with some one of the scale divisions. If the vernier be moved up along the scale from the position in the figure until the *first* vernier division coincides with the next scale division, namely 7.7, the zero will have been moved through a distance $\frac{1}{10}$ of the scale division, and the reading of the vernier, i.e., the position of its zero division, will be the scale reading plus $\frac{1}{10}$ of a scale division, i.e., $7.6 + 0.01 = 7.61$. If it be moved along until the eighth division of the vernier is in coincidence with a scale division, the reading of the vernier will be the scale reading plus $\frac{8}{10}$ of the smallest scale division, namely 7.68, and so on. *Hence to read any vernier, take the reading of the scale division next preceding the zero of the vernier and add m times the least count of the vernier, where m is the division of the vernier in coincidence with the scale.* Usually the divisions on the vernier are so numbered that this mental multiplication is unnecessary, the numbers giving directly the fraction of a division to be added. For example, in B the least count was found to be $\frac{1}{25}$ of 0.05 or 0.002, hence the fifth, tenth, fifteenth, etc., divisions of the vernier are numbered 1, 2, 3, etc., respectively, since a coincidence of these divisions with the scale means that the zero is $5 \times 0.002 = 0.01$,

$10 \times 0.002 = 0.02$, $15 \times 0.002 = 0.03$, etc., above the last scale division, respectively. As drawn, the reading of the zero is 30.026.

Verniers are generally so graduated that n divisions of the vernier equal $n - 1$ of the scale, as in A and B, in which case the vernier and scale graduations are numbered in the same direction. Sometimes however, it is convenient, for reasons of construction, to have the zero of the vernier at the top, in which case n vernier divisions are made to equal $n + 1$ of the scale. The vernier is then said to be "retrograde" and inspection of the second vernier provided will show that in this case the scale and vernier are graduated in opposite directions, and coincidences of the vernier are to be found by following along the scale backwards from the zero. The "least count" is found in the same way for both classes of verniers, as is evident from the above demonstration.

A convenient way of regarding a vernier, when its principle is once clearly understood, is to consider the whole vernier as a magnified smallest scale division, divided into n equal parts. A moment's consideration will show that, looked at from this point of view, the reading of the vernier is obtained by adding to the direct scale reading of the division next preceding the zero of the vernier, the direct reading of the coincident line of the vernier itself.

Procedure.—Become perfectly familiar with all of the verniers on the block provided. Set the verniers to read as follows: 1st, 8.03; 2d, 29.89; 3d, 30.874; 4th, 4° 10′; 5th, 0° 17′; 6th, 2° 58′ 30″; 7th, 48° 52′, and have them verified by an instructor. Then set them at random and record readings as they stand. These are also to be verified. Students should satisfy *themselves* that they thoroughly understand and can read *readily* all of the verniers provided before leaving the experiment, as similar verniers will be met with continually in subsequent work.

AREA MEASUREMENTS.

GENERAL DISCUSSION.

The methods for the measurement of areas such as occur in laboratory practice may be divided into two groups according as the area is a geometrical figure, or irregular in form. In the former case the measurements reduce themselves generally to the simple measurement of lengths from which the given area is computed by mensuration. Very large areas are of course measured by methods of triangulation, and are properly discussed in works on surveying.

By far the most common and important areas met with in the laboratory, such as indicator diagrams, hysteresis curves, alternating current wave forms, etc., are irregular in outline, and special methods and instruments have been devised for their measurement. The only method possessing high accuracy and employed to any extent is that in which a planimeter is used. This instrument is a form of mechanical integrator, by which the area of any figure is given at once by the simple operation of tracing its periphery with the point of the instrument. Various forms of planimeter have been invented, and practice with one of the best of them, the Amsler, is afforded in the following experiment. When, however, a planimeter is not available, one of the following methods may often be found of use:—

First.—The area may be computed by one of the following approximation formulæ, commonly called Simpson's rules. For the deduction of these rules, see Williamson's Integral Calculus, p. 212.

1. A series of equidistant parallel ordinates is drawn across the area. If the area of the figure be regarded as that of the polygon resulting from connecting the intersection of the ordinates with the curve by straight lines, the area will evidently be the *sum of one-half the extreme ordinates plus the whole of the intermediate ordinates, multiplied by the common interval.*

2. A closer approximation is obtained by assuming the curve to consist of a series of parabolic arcs, i.e., of second degree curves of the form $y = a + bx + cx^2$. Under this

assumption it can be shown that the area may be found as follows:—

Add together the first and last ordinate, twice every second intermediate ordinate, and four times each remaining ordinate, and multiply the sum by one-third the common interval.

3. A still closer approximation is obtained by supposing the curve to be made up of third degree elements of the form $y = a + bx + cx^2 + dx^3$.

Then if the number of intervals be taken a multiple of three, the area is obtained by the following rule:—

Add together the first and last ordinate, twice every third intermediate ordinate, and three times each remaining ordinate, and multiply the sum by $\frac{3}{8}$ the common interval.

It is evident that in all these cases the accuracy is increased by increasing the number of intervals.

Second.—By weighing.

Cut the area out of a piece of homogeneous cardboard, light bristol board or thin sheet metal, and weigh it. Cut out of the same material an accurately measured area, e.g., one square decimeter, or several square inches, and weigh it. The ratio of the two weights will evidently be the area desired, expressed in the unit of area adopted. The accuracy of this method depends on the homogeneity of the material weighed, and on the exactness with which the areas are cut out. This method is capable of an accuracy of about one per cent.

Third.—By coördinate paper.

Draw the area on accurately ruled coördinate paper. Count up the number of complete squares contained, and estimate the area of the remaining fractional squares. This method gives of course only approximate results.

Fourth.—In case of irregular polygons the total area may be divided into a number of triangles, the area of each of which may be computed from linear measurements.

Object.—The object of this experiment is to afford practice in the manipulation of the Amsler planimeter, to test the accuracy of the instrument, and to become familiar with the relative values of the units of area in the English and Metric systems. The experiment also gives practice in the use of the dividing engine.

Apparatus.—The type of Amsler planimeter provided is shown in Fig. 3. It consists of two arms hinged together, one of which is pivoted at the end by the needle point *F*, while the other, the tracer-arm, carries a point *P*, which is caused to trace the outline of the area to be measured. The planimeter rests at *D* on a roller-wheel so mounted on the tracer-arm that it slides without rotation when moved in the direction of the arm *A*, and rotates without sliding when moved at right angles to this direction. Motion in all other directions is accompanied by sliding and rotation. The number of complete revolutions made by the wheel *D* is recorded on the dial *G*, while fractions of a revolution may be noted by means of the vernier *E*, which reads to 0.001 of a revolution of *D*. The effective length of the arm *A* can be varied by clamping the sleeve carrying the hinge and recording mechanism at any desired position by the screw *S*. The exact adjustment at any desired division on *A* is made by the fine adjustment screw below *S*. The tracer-arm *A* is graduated at three points such that when the planimeter is set exactly at these positions, the instrument records areas in square decimeters, ten square inches or tenths of a square foot respectively. The instrument may thus be used to measure areas directly in any one of these units.[1]

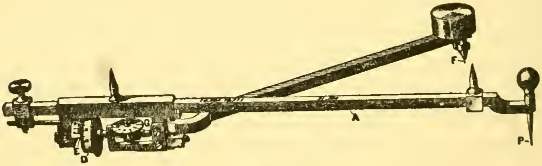


Fig. 3.

The planimeter is a delicate and expensive piece of apparatus and should be handled carefully. It should be wiped with chamois skin and returned to its case at the conclusion of the experiment.

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[1] For an elementary explanation of the theory of the instrument see Peabody's Manual of the Steam Engine Indicator, pp. 76–80. For the complete mathematical theory see Lanza's Dynamometers, Planimeters, Governors, and Fly-wheels, p. 28.

Procedure.—*First.*—To measure the area of a figure and to determine the experimental error involved.

Stretch tightly upon a drawing board a sheet of smooth unglazed paper. Tack upon this a sheet of laboratory note-paper, and with a pair of compasses draw in ink a circle of about one and a half inches radius. The line should be drawn very fine. Draw also several diameters extending across to the edge of the paper. Set the planimeter to read in square decimeters by unclamping S and moving the recording mechanism along the arm until the index on it nearly coincides with the division marked. Clamp S and set exactly on the division by means of the fine adjustment screw below S. Place the planimeter on the board with reference to the area to be measured as shown in Fig. 4, and see that the tracing point P can be moved freely around the whole circumference without cramping the instrument and without drawing the wheel over the edge of the paper on which the area is drawn. With a needle or pin prick a small

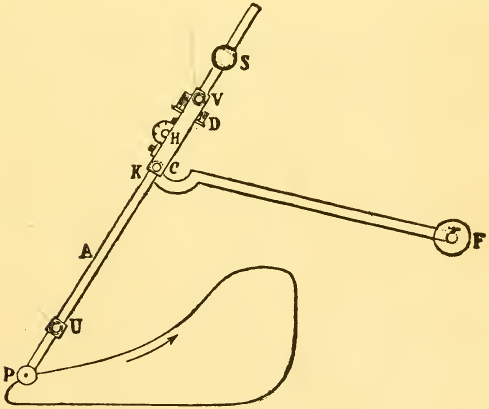

Fig. 4.

hole on the circumference as a starting point and move the tracer until it is exactly at this point. The recording wheel may now be adjusted so as to read exactly zero or, better, the reading may be taken on G and D as it stands. With P trace the circumference of the circle moving around clockwise, until the point P returns *exactly* to the starting point. This is of the greatest importance, for if the tracer does not start and end at exactly the same point a large error will be introduced. After a little practice it will be found that the tracer can be made to follow the line with surprising closeness. Positive errors introduced by running off outside the line tend to become eliminated by corresponding negative errors resulting from departing from the line on the other side, since both are likely to occur.

Planimeter the circle four times. Next set the planimeter to record in square inches, and planimeter the same area again four times. With the instrument still set to register square inches, planimeter the irregular area furnished by an instructor. Insert the figure and record its area in the notes. Measure the length of several diameters of the circle on the dividing engine, the manipulation of which will be explained by an instructor.

Second.—To determine the constant error of the planimeter.

The errors affecting planimeter measurements are of two kinds. First, indeterminate errors arising from the inability to exactly trace the area owing to the unsteadiness of the hand, slip of the revolving wheel, etc.; and, second, constant errors arising from inaccurate adjustment or construction of the instrument itself. The former errors should not exceed 0.01 or 0.02 square inch; and the latter, in instruments of the best construction, may be made negligible, i.e., less than the smallest area which the instrument is capable of recording.

For the purpose of determining the instrumental error, a geometrical area (circle or square) is engraved on a metal plate, and its linear dimensions measured with great accuracy on a dividing engine. The computed area is then compared with the area as determined by the planimeter. If the figure is slightly cut into the metal, the tracer of the planimeter can be made to follow the area exactly, and the indeterminate errors due to trembling of the hand, etc., are thus eliminated.

Planimeter the circle on the test plate provided, in square inches, and measure its diameter on the dividing engine.

Computation.—*a.* Compute the mean, *a.d.*, *A.D.*, and the percentage deviation of a single observation and of the mean, of the two series of observations on the area of the circle. Compare the mean value of the area expressed in square inches with the area calculated from the measurements of the diameter of the circle. What is the percentage difference? How many figures should be retained in π in the computation?

b. From the mean value of the area expressed in square inches and square decimeters respectively, compute the value of one square decimeter in square inches, and conversely.

c. Give the area of the irregular figure provided.

d. From a comparison of the observed and computed area of the test plate, calculate the error of the planimeter per square inch.

Precision Discussion.—Suppose it is desired to construct a circular test plate for calibrating a planimeter which shall fulfil the following conditions: first, its true area must be known to 0.1 per cent, and second, it must be sufficiently large so that the smallest value which the planimeter is capable of recording, namely, that corresponding to one vernier division, i.e., 0.01 square inch shall not be greater than 0.1 per cent of the whole area. What must be the diameter of this circle and how closely should it be measured?

If the allowable deviation in the measured area is to be less than 0.01 square inch, and this not greater than 0.1 per cent of the total area, the area of the circle should be not less than 10 square inches, i.e., the diameter must be

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{40}{\pi}} = 3.6''$$

The value of the diameter must be known to ½ of 0.1 per cent = 0.05 per cent, since $\frac{\Delta}{A} = 2\frac{\delta}{d}$, i.e., the fractional error in the area is twice as great as the fractional error in the diameter. Hence d must be measured to 0.05 per cent of 3.6 inches = 0.0018 inch.

Problem.—What would be the numerical and percentage error in an area of 8.315 square inches measured with the planimeter used if the correction for the constant error of the instrument, as above determined, were not applied?

MEASUREMENT OF THICKNESS.

THE MICROMETER CALIPER, SPHEROMETER, AND OPTICAL MICROMETER.

Object.—The object of this experiment is to give facility in the use of the micrometer caliper, spherometer, and optical micrometer, instruments which are designed to measure the thickness of small objects with accuracy. The discussion of the results illustrates the effect of various sources of error, and affords practice in "weighting" observations.

MICROMETER CALIPER.

Apparatus.—This instrument is shown in Fig. 5. It consists of a uniform screw, one end of which is fixed to the inner end of a movable hollow barrel *B*, by means of which it can be turned back and forth in a nut *N*, rigidly attached to the body of the instrument. The nut *N* is graduated to correspond with the pitch of the screw so that one revolution of the screw advances *B* through just one (or sometimes one-half) division on *N*. Fractions of a revolution are determined by the graduations on the edge of the barrel *B*.

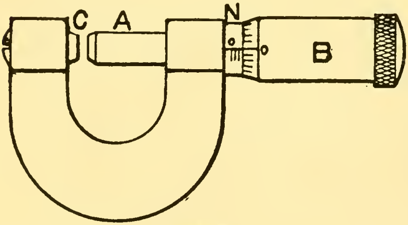

Fig. 5.

The end of the screw *A* is finished with a polished plane face at right angles to its length. A similar fixed plane face is attached to the instrument at *C*. These faces should be plane and parallel so that perfect contact is made between them when screwed together. *C* is so adjusted that when the end of the screw *A* (called its tooth) is in contact with *C* the zero graduation on *B* stands at zero on *N*. Often this adjustment is not made exactly, in which case the instrument has a "zero error." The difference in reading of the instrument when *A* and *C* are in contact (zero reading) and when some object is placed between them gives at once the thickness of the object.

Procedure.—Determine the "zero error" of the calipers (four settings will suffice) and then measure the thickness of the object provided, taking nine independent settings at different parts of the object to insure a measure of its average thickness. In setting the caliper do not strain it by turning the screw so as to clamp the jaws too tightly. It is best to turn the screw until the fingers just slip off the end of the barrel.

THE SPHEROMETER.

Apparatus.—Spherometers are instruments designed to measure the thickness of small objects with a high degree of precision and to measure the radius of curvature of spherical convex or concave surfaces. It is from this latter use that the instrument derives its name.

One form of this instrument (due to Perreaux) is shown in Fig. 6. It consists essentially of a vertical micrometer-screw moving in a nut fixed at the centre of an equilateral tripod. The head of the micrometer screw *A* is a graduated circle, the position of which is read on a fixed vertical scale *B* graduated in divisions equal to the pitch of the screw. One complete revolution of the screw thus raises or lowers *A* through one division of the fixed scale. Fractions of a revolution are read off on the head of the micrometer with reference to this scale.

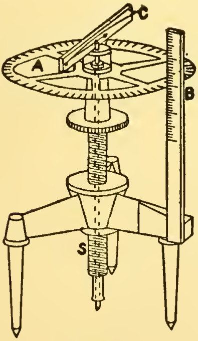

Fig. 6.

The sensitiveness of the instrument

depends on the precision with which one can detect when the end of the micrometer screw just makes proper "contact" with the object placed beneath it. In the ordinary form where no special device is provided for facilitating this observation, the screw is turned until the instrument just turns as a whole on the end of the screw as a pivot, or until the hand can just detect a slight rocking motion of the instrument like that of a table, one of whose legs is too long. With these apparently somewhat crude methods of detecting contact, settings to one or two thousandths of a millimeter may readily be made.

To increase the sensitiveness of the instrument, various devices have been invented. The Perreaux multiple lever attachment is that shown in the figure. The micrometer screw is hollow and through it a steel rod passes freely. The upper end of this rod acts upon a system of two multiplying levers, by means of which a slight motion of the rod becomes greatly magnified. *C* is a fixed reference edge, to which the end of the longer lever may be brought, in which case the pressure exerted on the lower end of the rod will evidently be the same for different settings. In this instrument the pitch of the screw is 0.5 mm., the head is graduated into 500 parts, and hence reads directly to 0.001 mm., and by estimation to 0.0001 mm.

Procedure.—Measure the thickness of the object previously used as follows: Place the spherometer on a plate glass surface, and turn the screw until the rod passing through it presses upon the plate just sufficiently to bring the end of the long lever up to the reference mark *C*. Record the position of the head of the micrometer on the vertical scale, estimating by eye, to tenths of the smallest division. Move the spherometer to a new position on the plate and set again. Proceeding in this manner, take nine independent settings over the plate. Next turn the micrometer back through a distance somewhat greater than the thickness of the object to be measured and place the object upon the plate under the micrometer screw. Make nine independent settings as before on different parts of the surface. The difference of the means of these two series of settings gives the desired thickness.

Measurement of the radius of curvature of a spherical surface by means of a spherometer.—This use of the instrument may be deferred until the experiment on the focal length of lenses is performed.

To measure the curvature of a spherical surface with a spherometer, three measurements are necessary. The spherometer is first placed on a plane surface, and the micrometer screw turned until it just makes contact with the surface. Let this reading be denoted by m_1. The instrument is then placed on the surface or lens to be measured, and the micrometer turned until it again touches the surface with the same pressure as before. Let the reading be denoted by m_2. Then $m_2 - m_1 = h$ is the height of a segment of the sphere whose radius R is to be determined, the base of the segment being a circle passing through the three feet of the spherometer. If r is the radius of this circle, i.e., the distance of the end of the micrometer screw from the feet of the spherometer when all four points lie in the same plane, then

$$R = \frac{r^2}{2h} + \frac{h}{2}.$$

To measure r, make a slight imprint of the ends of the legs of the spherometer and of the central screw on a sheet of paper. Measure the distance of each leg from the axis of the screw, and take the mean; r may also be computed by geometry from the mean distance between the legs of the instrument, since these form an equilateral triangle.

THE OPTICAL MICROMETER.

Apparatus.—This instrument is shown in Fig. 7. It consists of a simple micrometer screw of 0.5 millimeter pitch, the head divided into 250 parts, thus reading directly to 0.002 mm. It is used for thickness measurements only. The unique feature in its construction is the beautiful device

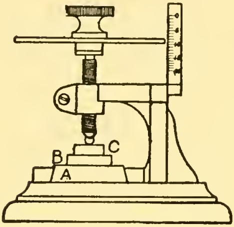

Fig. 7.

for indicating the moment of "contact" of the end of the screw with the object. This is effected as follows: The base of the instrument is a plate of plane black glass, *A*. On this rests a small plate of glass, *B*, the faces of which are approximately plane and parallel. If this plate be pressed upon the glass base, care being taken that no dust particles are between them, Newton's interference fringes will be produced by the thin air film between the plates. The fringes will appear colored if white light be allowed to fall on the plate, and yellow, if sodium light be used. If, now, the micrometer screw be turned slowly down, the instant that it comes into contact with the plate, a sudden displacement of the fringes will be observed. The instant of contact can thus be detected with the greatest nicety.

Procedure.—Wipe the glass base and plate very carefully and place the latter under the micrometer screw so that the fringes are clearly seen. Monochromatic light is best, but not at all necessary for observing the phenomenon. Turn the screw until it just makes contact with the plate, as indicated by the displacement of the fringes, and record the reading of the micrometer head. Make, at different portions of the plate, nine settings. Place the object *C*, whose thickness is to be measured, between the end of the screw and the test plate, and take a second series of settings as before. The difference of the means gives the thickness of the object.

Computation.—Compute the thickness of the object from the measurements made by each of the three instruments. Compute also the deviation measure of each result. (See Notes on Precision of Measurements.)

Questions and Problems.—1. From a comparison of the results would you say that any of the instruments had a constant instrumental error, and why?

2 (*Optional*).—If the deviation measures of the final results obtained with the spherometer and optical micrometer be assumed equal to their precision measures, compute their relative "weights" and their "weighted mean." (See Precision of Measurements, p. 20.)

THE SPIRIT LEVEL.

PART I.—ADJUSTMENT OF LEVEL.

Object.—To adjust a spirit level and to level a surface.

Discussion.—The spirit level is an instrument used for rendering surfaces and parts of apparatus truly horizontal, that is, perpendicular to the direction of the earth's attraction, or to a plumb line set up at the place in question. It consists essentially of a glass tube, *A*, the upper surface of which is slightly curved in such a manner that a longitudinal section (through a theoretically perfect level) would be the arc of a circle of several feet radius. This is effected by carefully grinding the inner surface to the arc of a circle. The greater the radius of curvature, the more sensitive is the level. The tube is filled, except for a small air bubble, with alcohol or a mixture of alcohol and a little sulphuric ether. The latter renders the liquid more mobile and the bubble more reliable and rapid in its action. Pure ether cannot be used on account of its great expansibility, unless the tube is provided at one end with a suitable expansion chamber. Such levels are used only for astronomical or geodetic work where the greatest possible sensitiveness is desired. The tube is firmly mounted in a brass case to protect it from injury, the upper surface of the level alone being exposed to view. This surface is graduated to the right and left of the

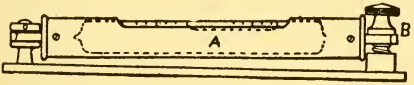

Fig. 8.

centre in equidistant divisions. The level in its case is mounted on either a flat or on a ⋏-shaped base according as it is to be used for levelling plane or cylindrical surfaces, such as the telescopes of various optical instruments. The case containing the level is provided with an adjusting screw, *B*, at one end (sometimes at both), so that it may be brought into such a position that the tangent plane through the centre of graduation is parallel to the plane of the base, or, in other words, so that when the level is placed on a horizontal surface, both ends of the bubble will be equally distant from the centre. That this condition must be fulfilled in an adjusted level is evident, since the bubble always assumes such a position that the free surface of the liquid is truly horizontal, that is, perpendicular to the direction of the earth's attraction; hence the plane of the base and the tangent plane through the centre of graduation must be parallel, if when the former is horizontal, the bubble shall take up a symmetrical position with respect to the latter.

A level should never be assumed to be in adjustment without testing. This may be done, of course, by placing the level on a horizontal surface, and noting whether the ends of the bubble are at equal distances from the centre of the graduations; if they are not, the adjustment would consist in simply turning the screw *B* until this was the case. As a horizontal surface is seldom at one's disposal, this procedure is usually impracticable, and the adjustment to be made depends on the following demonstration.

Demonstration.—Let *XY* represent the plane of the base of the level, which may or may not be horizontal; we will assume that it is not. Suppose first, that the level is in adjustment, i.e., that *AB*, the direction of the tangent plane at the centre *o* of the graduations, is parallel to *XY*. The bubble will then take up a position *a b* depending on the inclination of *XY* to the horizontal. If the level be now exactly reversed, *A* will coincide with

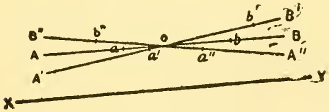

Fig. 9.

the former position of B relatively to the plane XY, and the bubble will take up the same position relative to the surface as before; in other words, the end of the bubble towards A will occupy exactly the same position as that previously occupied by the end towards B, and if the new readings be called a_2, b_2, we shall have $a = b_2$ and $b = a_2$. This, then, is the condition which must be fulfilled when the level is in adjustment. If, now, the plane AB is not parallel to XY but is inclined to it as $A'B'$, the bubble will take up a higher position in the tube a' b'. On now reversing the level, the bubble will again move to the higher end of the tube and assume the position $a''b''$. In this case a' will no longer coincide with b'' nor b' with a''; the difference $b' - a''$ or $b'' - a'$ is proportional to the amount the level is out of adjustment. If, therefore, the adjusting screw be turned so that the bubble moves by one-half this difference, the level should be in adjustment. Several trials are usually necessary if the level is very sensitive.

Apparatus.—The apparatus required for adjusting a level besides the level itself is a plane surface mounted on a plate provided with three levelling screws. A carefully ground plate of glass about three-quarters of an inch thick is very satisfactory for this purpose. The ends of the levelling screws should set in conical supports to prevent any lateral motion of the plate. The linear dimensions of the surface should be somewhat greater than the length of the level, as the latter must never be placed on the plate with one end projecting over the edge. The plate should be placed on a shelf or support as free from vibration as possible.

Procedure.—*First.*—Taking the level and plate as they stand, both presumably much out of adjustment, the first step is to render the plate approximately horizontal so that both ends of the bubble can be read on reversal. To do this, place the level on the plate parallel to two levelling screws. In general, one end of the bubble will disappear under the end of the level. Note the approximate position of the bubble. Mark with a pencil the exact position of the level on the plate and reverse. The bubble will either remain at the same (absolute) end of the level, in which case the plate is approximately horizontal,—since the bubble of an unadjusted level will remain at rest in all positions of the level on a horizontal surface,—or it will move across to the opposite end of the level, in which case the plate is more out of adjustment than the level. If the former is found to be the case, the adjusting screw of the level should be turned so that the bubble moves out toward the centre. If the latter is the case, the same result should be obtained by turning the levelling screw of the plate. Note the new position of the bubble and reverse again. Repeat the adjustment of the level or plate according as the bubble remains nearly at rest or moves across from one end of the level to the other. A few trials will bring the plate and level into such adjustment that both ends of the bubble will fall on the scale in the direct and reversed positions. No readings need be recorded in this preliminary adjustment. When the adjustment is made, however, read and record the exact positions a' b' of both ends of the bubble to tenths of a division, always recording the readings toward the A end as a's, and those toward the B end as b's. Reverse the level exactly and call the new readings a'' and b''. If a' does not equal b'' and $b' = a''$, adjust by half the difference. Record the new readings again under a' and b' and test the adjustment by reversal. If it has been correctly made, $a' = b''$, $b' = a''$; if not, repeat until this is the case. An adjustment correct to 0.2 of a division will be considered satisfactory with the levels used. This corresponds to about 5″ of arc and a finer adjustment often requires an excessive expenditure of time.

The same procedure described above applies equally well to the adjustment of levels parallel to the axis of reading telescopes of cathetometers, spectrometers, etc.

In all manipulations with a sensitive level, both in adjusting and in determining the angular value of a division as described below, care must be taken to avoid heating the level by the hand, breath, and in case of very sensitive levels, by radiation from the body, as a variation of temperature causes a change in the length of the bubble.

Second.—With the now adjusted level, level the plate. This is best done by placing the level on the plate, first, parallel to two of the three screws, and turning one of them until the ends of the bubble are equidistant from the centre, and second, at right angles to this position and levelling by the third screw. If two lines at right angles to each other in the same plane are horizontal, the plane itself is also horizontal. Many instruments are provided with two adjusted levels fixed permanently at right angles to each other on the base of the instrument. In this case, the instrument is levelled by simply turning the levelling screws until each bubble stands at the centre of its respective level.

PART II.—SENSITIVENESS OF LEVEL.

Object.—To determine the angular value of a level division and the radius of curvature of the level.

Discussion.—Although the principal use of a level is to render instruments horizontal, it is also used in astronomy and geodesy for measuring very small vertical angles. For this purpose, it is necessary to know the angular value of the level graduations, that is, the angle through which the level must be turned to cause the bubble to move over one division. This angle defines the sensitiveness of the level, and, in a perfect level, should be the same for all divisions. This, however, is seldom the case, owing to slight irregularities in the grinding of the level. For accurate angular measurements, the value of each division must, therefore, be determined.

It is of the greatest importance that the levels accompanying any instrument be of a sensitiveness corresponding to the grade of work to which the instrument is to be put, and comparable with the degree of accuracy attainable with the other adjustments and reading parts of the instrument. A too sensitive level used in conjunction with an instrument of medium precision, or with one set up in an unsteady position, is almost useless, while to use a level capable of indicating only several minutes of arc on an instrument, the circle of which reads to thirty seconds, is to sacrifice completely the precision of the instrument.

Apparatus.—For determining the angular value of a level division, a level tester is employed. This consists of a massive bar of iron, one end of which *A* rests on two projecting steel points, while the other end *B* carries a micrometer screw, the point of which rests on a polished piece of metal or glass. By means of this screw, the end *B* can be raised or lowered any desired amount. The tester sets on a heavy metal plate *C*, which should be mounted on a rigid wall or shelf as free from vibration and jarring as possible. The micrometer screw of the instruments provided is of one millimeter pitch and the head is divided into 100 parts, thus permitting thousandths of a millimeter to be obtained by estimation.

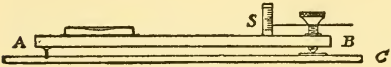

Fig. 10.

Procedure.—Place the level on the bar, preferably near the end, and turn the micrometer screw until one end of the bubble is brought exactly to the first division. Record the reading of the micrometer. Repeat the setting four times, recording the micrometer reading each time. In these settings the bubble should always be brought up to the division in the *same direction.* This precaution is necessary in order to eliminate errors arising from tardy action of the bubble, due to friction against the tube, and to any backlash there may be in the micrometer screw. In the same way, take a series of four settings each on divisions 2, 3, 4, and 5 of the level. Finally remove the level and measure with a steel scale the distance between the point of the micrometer screw and the line connecting the steel points on which the bar rests. Measure also the length of the graduations on the level itself. This is best done by marking the total length of four consecutive divisions on a strip of paper and measuring the distance between these marks with a scale; if greater precision is desired, a dividing engine or two microscopes may be used. This last measurement is necessary for computing the radius of curvature of the level.

Computation.—From the mean values of the settings taken, compute the *average* amount the level tester is raised to move the bubble over one division. With this and the length of the tester compute the corresponding angular value in seconds. From the average length of the graduations on the level compute its radius of curvature.

Answer the following questions in a general way by considering the percentage uncertainty in each of the quantities measured.

Questions.—1. With what precision must the length of the tester be measured in order that the deviation in this measurement may be negligible in the computed value of the angle?

2. About how precise do you estimate the value of the computed radius of curvature to be, and why?

3. Does this precision represent the *accuracy* of the result, and why?

MEASUREMENT OF MASS.

GENERAL DISCUSSION.

The determination of the weight or the mass of a body, one of the most frequent and most fundamental of all physical measurements, is almost invariably made with an equal arm balance, and the construction of this instrument has reached a very high degree of perfection. Balances suitable for analytical and most physical work are sensitive to at least 0.1 milligram under a load of 100 grams, i.e., to 1 part in one million, and in some very delicate balances a considerably greater precision is attained.

The following description applies, with slight modifications in details, to the several forms of balance used in the laboratory. The essential parts of a good equal arm balance are shown in Fig. 11.

The balance beam, constructed to combine maximum rigidity with minimum weight, carries at its centre a knife edge *A*, which, when the balance is in use is lowered upon

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[1] One radian = 206265″.

a plane on which the beam swings. At the ends of the beam are fixed two other knife edges, *B*, *B*, upon which the scale pans are suspended by means of planes. Knife edges and planes are constructed of agate in the best balances. In cheaper instruments they are made of steel. These three knife edges are adjusted so as to lie very nearly in the same straight line. By the length of the balance arms is meant the distance of each end knife edge from the central one, i.e., the distance *AB*. Both arms should be made as nearly equal in length as possible, and in good balances a difference not more than one part in 50,000 or one part in 100,000 is attained. At one end (sometimes both ends) of the beam is a fine screw, provided with a small nut, *N*, by means of which the effective weights of the balance arms may be adjusted equal. The beam is provided with a long pointer swinging over a fixed scale, *S*, near the base of the balance case. A small adjustable nut, *M*, on the pointer serves to raise or lower the centre of gravity of the moving parts of the beam, an adjustment which is necessary for reasons pointed out below. In some balances this adjustment is made by a nut attached above the beam.

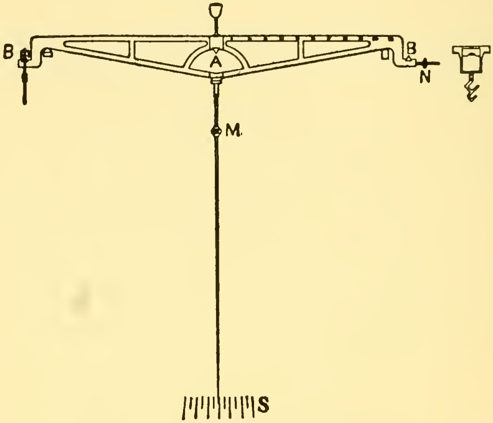

Fig. 11.

When not in use, the beam should always be raised off its support, and the pans off their respective knife edges, to prevent dulling the knife edges by jarring. This is effected by means of a frame operated by a milled head at the front of the balance case. This mechanism for raising and lowering the beam should be so constructed that no lateral displacement of the knife edge on its support can occur. To further provide against injury from jarring when lowering the beam and pans, and to prevent the pans from swinging, light stops or rests are provided beneath them. These should always be raised to support the pans, except when a weighing is being made; they are then carefully lowered by pressing the button operating them at the front of the case. The height of these rests is regulated by means of an adjusting screw at the back of the balance. A good balance is also provided with either a plumb bob, or preferably, a spirit level, and with levelling screws.

Theory of the Balance.—Reduced to its elements, an equal arm balance consists of a rigid beam supported horizontally at its centre on a knife edge and loaded at its ends. The various conditions upon which the sensitiveness of a balance depends will be readily seen from the following demonstration:

Let $l =$ length of arms (assumed equal).
$w =$ weight of beam acting downward through centre of gravity.
$l' =$ distance of centre of gravity, CG, below point of support of the beam.

With a load of P grams in each pan, the balance will evidently be in equilibrium. If a small additional weight p be added to the right-hand pan, the balance will swing through an angle α and take up a new position of equilibrium as in Fig. 12. The condition for equilibrium will evidently be found by taking moments of the three effective forces, P, $P + p$, and w about an axis passing through the knife edge, i.e.,

$$Pl \cos \alpha + wl' \sin \alpha = (P + p)\, l \cos \alpha,$$

or

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha} = \frac{lp}{wl'}. \tag{1}$$

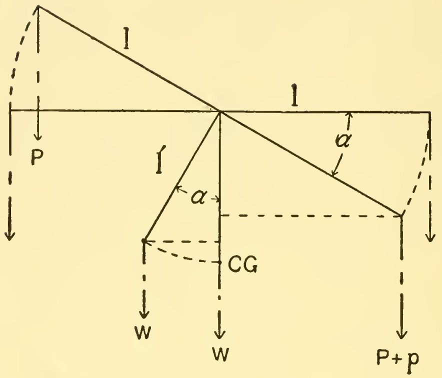

Fig. 12.

Now the *sensitiveness* of a balance is measured by the angle through which the beam will rotate when a definite weight, usually one milligram, is added to either pan. As the angle practically coincides with its tangent for very small angles, the expression deduced for tan a may be taken as a measure of the sensitiveness of the balance when $p = 1$ milligram.

From this it follows that the sensitiveness of a balance is directly proportional to the length of the balance arms; inversely proportional to the weight of the beam; and inversely proportional to the distance of the centre of gravity below the point of support. It is also evident that the observed scale deflection is proportional to the length of the pointer.

Unfortunately the above conditions for maximum sensitiveness conflict with each other in the mechanical construction of a balance. Thus long arms are incompatible with minimum weight. The length of the arms is also practically limited by another condition, namely, the time of swing of the balance, which must not be excessive. Regarding a balance as a compound pendulum, it can be shown that its time of vibration is given by the expression

$$t = \pi \sqrt{\frac{wk^2 + 2Pl^2}{wl' g}} \qquad (2)$$

where

k = radius of gyration,
g = acceleration due to gravity, and w, P, l, and l' have the same significance as before.

From this it will be seen that a limit is soon reached beyond which it is undesirable to increase the sensitiveness of a balance by increasing the length of its arms, l, for its time of vibration becomes excessive, and the process of weighing very tedious. A period of 12 to 15 seconds is about the maximum time desirable. Again the sensitiveness cannot be increased by diminishing l' beyond a certain limit for the same reason. (The centre of gravity, CG, must, of course, always be below the knife edge, otherwise the balance will be in neutral or unstable equilibrium). Makers have, therefore, a considerable choice of conditions which they can vary to obtain a given result, and we find on the market excellent "Long Arm" as well as "Short Arm" balances. Long arm balances, although sensitive, possess the disadvantage of a long time of swing (10 to 15 seconds) which renders weighing with them very tedious. For moderate loads short arm balances constructed with light beams (aluminum or aluminum alloys are employed) are much to be preferred, as they are not only sensitive, but very quick in their action (time of vibration 6 to 10 seconds).

In the above discussion the assumption is made that the points of support of the pans and the knife edge about which the beam swings are in the same straight line. This is an ideal condition which is always departed from more or less in actual instruments, since no beam can be made absolutely rigid, and, hence, if the above condition be fulfilled for some one load, it must be further and further departed from with increasing load. To determine the effect of this deviation from the ideal condition, suppose that the lines joining the points of support of the pans with the central knife edge make an angle β with the horizontal line through the latter. (See Fig. 13.) When the balance is in equilibrium under loads P and $P + p$ we shall have

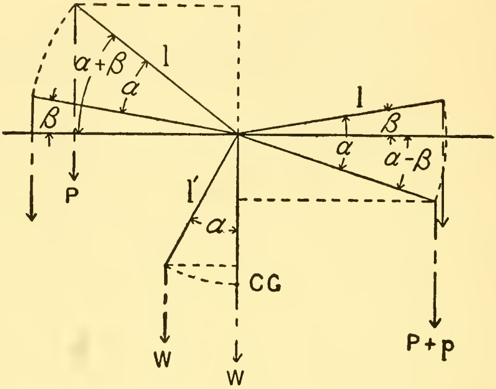

Fig. 13.

$$Pl \cos (\alpha + \beta) + wl' \sin \alpha = (P + p)\, l \cos (\alpha - \beta)$$

or expanding and solving for tan α we obtain

$$\tan \alpha = \frac{1}{\dfrac{wl'}{pl} \cdot \sec \beta - \dfrac{2P + p}{p} \cdot \tan \beta} . \tag{3}$$

If the points of support of the pans are *below* the central knife edge, β becomes negative, and for this case

$$\tan \alpha = \frac{1}{\dfrac{wl'}{pl} \cdot \sec \beta + \dfrac{2\,P + p}{p} \cdot \tan \beta} . \tag{4}$$

The value of β which renders both of these expressions for the sensitiveness a maximum is easily seen to be $\beta = 0$, in which case both formulæ reduce to $\tan \alpha = \dfrac{pl}{wl'}$, as already shown. If the points of support of the pans under zero load are above the horizontal plane through the central knife edge, the sensitiveness of the balance will increase with increasing load. If the points of support of the pans are below the horizontal through the central knife edge, the sensitiveness will continually diminish with increasing load. A determination of the sensitiveness of a balance under different loads affords, therefore, information regarding this point in the construction of the beam. In the best construction the beam is so designed that the maximum sensitiveness is reached at about one-half the maximum load which the balance is intended to carry.

It will be seen from (3) that in the case when the points of support of the pans are above the central knife edge it would be theoretically possible to increase the load, P, to such a value that $\frac{2P+p}{p}\tan\beta=\frac{wl'}{pl}\cdot\sec\beta$, in which case the balance would reach a state of unstable equilibrium. The period of vibration of the balance would at the same time be greatly increased.

PRECAUTIONS TO BE OBSERVED IN USING A BALANCE.

1. The balance should be set up on a firm support fixed to a wall or on a pier which is as free from mechanical vibration as possible. A place should be chosen where direct sunlight will not fall upon the balance, as the resulting temperature changes will introduce irregularities and errors in the weighings.

2. To prevent the knife edges (if of steel) and other parts of the balance from tarnishing, an open dish of some drying agent, as concentrated sulphuric acid, may advantageously be kept in the balance case.

3. When not in use, the balance beam should always be raised off the knife edges. Injury to the knife edges from jarring will otherwise result.

4. The beam should always be lowered slowly and carefully. When lowered, the pans should just rest upon the stops or arrests beneath them, and the index should stand at the middle of the scale. The pan arrests when lowered should free both pans simultaneously.

5. The beam should never rest upon the knife edge while weights or substances are being added to or removed from the pans, except in the case of small fractional weights less than one gram.

6. The beam should be set swinging either by dropping the rider upon the beam or removing it for a moment, or by fanning one pan gently with a motion of the hand. Never hit the pan with the forceps, or attempt to set the beam swinging by suddenly lowering it upon its knife edge.

7. All weighings should be made methodically, not haphazard; otherwise an enormous loss of time will result. A little experience is necessary before rapid weighings can be made.

8. Final adjustment with the rider and all observations of the swing of the index should always be made with the balance case closed, to prevent errors arising from air currents.

9. The weight of a substance should be recorded, *first*, by adding up the weights missing from the box (in which every weight should have its own place), and *second*, by adding the weights as they are returned to the box. This gives a check which is most desirable, as nothing is easier after having made a correct weighing than to record the weight erroneously.

10. Any substance which can possibly injure the scale pans should never be placed directly upon them, but upon a watch glass or in some other suitable vessel.

11. Substances should never be weighed while hot, as air currents will be produced which will introduce serious errors in the weighing.

12. Care should be taken never to overload a balance by attempting to weigh bodies heavier than the balance is constructed to carry.

METHODS OF WEIGHING.

The weight of a substance may be obtained with an equal arm balance by any one of the following methods, the relative advantages of each of which are stated below.

First.—Ordinary Method or Method of Equal Swings. In this method the balance must first be put in perfect adjustment, so that under zero load the index swings equal distances to the right and left of the middle scale division, allowance being made for the diminution in amplitude of successive swings resulting from friction at the knife edges, air resistance, etc. It is convenient to call the middle division of the scale, zero, and to reckon deflections to the right and left of it. The body to be weighed is placed in the left-hand pan and weights added to the right-hand pan until, with a final adjustment of the rider, the pointer swings equal distances to the right and left of the middle division. This method is applicable only when the balance is provided with a rider. It is not to be recommended when the extreme precision attainable with the balance is desired. For ordinary weighing it is, however, the method commonly used.

Second.—Method of Swings. In this method the balance need not be initially adjusted to swing equally to the right and left of the zero under no load, and it is the only method applicable to balances not provided with riders. The procedure is as follows: Determine the point of rest under zero load by noting the position of the extreme swing of the pointer, *three times to one side and twice to the other side of the position of rest.* In this case the divisions on the scale should be numbered from right to left, the middle division being thus numbered 10 instead of 0. Suppose the readings are as follows:—

Left.	*Right*
14.3	4.2
14.1	4.4
	4.5
14.20	4.37
	14.20
	2)18.57

Point of rest = 9.29

The point of rest will then evidently be 9.29. An odd number of readings is always taken to eliminate the effect of air resistance.

Call the position of rest under zero load as thus found a_0. Next, nearly balance the body by a weight w_1, and determine the point of rest a_1 as before. (If the balance is provided with a rider, the weight w_1 should be adjusted to the nearest 0.001 gram.) Finally add a weight m grams (0.001 gram or 0.002 gram if a rider is provided), so that the body is *slightly* over balanced, and again find the point of rest a_2 under the weight $w_2 = w_1 + m$. As the point of rest shifts proportional to the change in load, *i.e.*, as

$$\frac{w - w_1}{a_0 - a_1} = \frac{w_2 - w_1}{a_2 - a_1} = \frac{w_1 + m - w_1}{a_2 - a_1},$$

we obtain for w, the true weight which would bring the point of rest back to a_0,

$$w = w_1 + m\frac{a_0 - a_1}{a_2 - a_1}.$$

For example, if $a_0 = 9.29$; $a_1 = 8.27$ under a load $w_1 =$ 14.167 gms.; $a_2 = 10.41$ under a load $w_2 = 14.168$ gms., then

$$w = 14.167 + 0.0010\frac{9.29 - 8.27}{10.41 - 8.27} = 14.167 + 0.00048 =$$ 14.16748 gms.

This method requires an accurate knowledge of the point of rest under zero load. As this is likely to vary if the balance is jarred or its level changed, it must be frequently determined, unless the balance is set up under exceptionally steady and isolated conditions. When a large number of weighings are to be made, and the extreme precision of the balance is desired, this method is to be preferred to the preceding.

Double Weighing.—Both of the preceding methods are subject to constant instrumental errors arising from inequality in the length of the balance arms. This may be corrected for by a determination of the ratio of the length of the arms (see p. 43), or by either of the two following methods of so-called "*double weighing.*"

Third.—Borda's Method or Method of Taring. The body to be weighed is first balanced or "tared" by an appropriate substance, e.g., shot or fine sand. It is then removed from the pan and weights added in its place, until the taring substance is exactly balanced. The weight added is then evidently equal to the weight of the substance whatever be the relative length of the balance arms.

Fourth.—Gauss's Method. The substance is first placed in the left-hand pan and weighed by either of the first two methods. Let the weight thus obtained be w_r. It is then placed in the right-hand pan and weighed again, the weights being added in this case to the left pan. Call the weight thus obtained w_l. If r and l be the lengths of the right and left-hand arms of the beam, respectively, and w the true weight of the body, then

$$w\,l = w_r\,r$$

$$w_l\,l = w\,r$$

whence eliminating r and l,

$$w = \sqrt{w_r w_l}\,.$$

Since w_r and w_l usually differ from each other at most by only a very small amount, the arithmetical mean may be taken equal to the geometrical mean without introducing a sensible error, i.e.,

$$w = \tfrac{1}{2}(w_r + w_l\,),$$

thus simplifying greatly the computation. If, for example, the substance weighs 1 gram and the weights found are $w_r = 1.001$ and $w_l = 1.000$, then $\frac{1}{2}(w_r + w_l) = 1.0005$, while $\sqrt{w_r\ w_l} = 1.0004999$.

Of these two methods of double weighing, Gauss's Method is to be preferred, as the two weighings afford a check on each other, and the method is, on the whole, the more rapid of the two.

Reduction of apparent weight of a body to its true weight in vacuo.—The weight of a substance as determined by an equal arm balance, after all corrections have been made for instrumental errors, is only its apparent weight under the given conditions of temperature and pressure at the time of weighing. This arises from the fact that, in general, the volumes of air displaced by the body and by the weights are different, owing to the difference in their densities. The buoyant effect of the displaced air is, therefore, different on the body and on the weights, causing the former to apparently weigh too much if it occupies a smaller volume than the weights, and too little, if the converse is the case. Only when the substance weighed has the same density as the weights, is its apparent weight the same as its true weight in vacuo.

Let W = true weight of substance in vacuo.
w = apparent weight.
δ = specific gravity of the substance.[1]
Δ = " " " " weights.[1]
σ = weight of 1 cc. of air under the conditions of temperature, pressure, and humidity at the time of weighing.

Then evidently the body is buoyed up by a weight of air $\frac{W}{\delta}\sigma$ and the weights by the amount $\frac{w}{\Delta}\sigma$. We have, therefore, when the substance and weights are in equilibrium,

$$W - \frac{W}{\delta}\sigma = w - \frac{w}{\Delta}\sigma,$$

whence
$$W = w\frac{1 - \frac{\sigma}{\Delta}}{1 - \frac{\sigma}{\delta}},$$

or
$$W = w\left[1 + \sigma\left(\frac{1}{\delta} - \frac{1}{\Delta}\right)\right] \text{ approximately (1).}$$

By this important formula the true weight in vacuo of any substance may be calculated from its apparent weight in air.

That the value of this correction may be very appreciable when δ and Δ differ widely, may be seen from the following table which contains the value of the correction term for several common substances. The correction is computed in *milligrams* for one gram of substance, assuming $\sigma = 0.0012$, its mean value for ordinary atmospheric conditions in the laboratory, and $\Delta = 8.5$ (Becker's brass weights).

---

[1] Note: In general δ and Δ need be known only approximately.

Substance.	δ	$\sigma\left(\frac{1}{\delta} - \frac{1}{\Delta}\right)$ *in milligrams.*
Alcohol	0.8	+ 1.36
Water	1.0	+ 1.06
Glass	2.5	+ 0.34
Brass	8.5	0.00
Mercury	13.6	— 0.055
Platinum	21.4	— 0.085

Thus a mass of water weighing exactly one gram in air would weigh 1.00106 grams in vacuo. Neglect to apply this correction in the case of water would, therefore, introduce an error of 0.1 per cent. In the case of glass the error would be 0.03 per cent, and so on.

When a precision not exceeding 0.1 per cent is desired, the correction is evidently negligible for substances heavier than water. When, on the other hand, substances like gases or vapors are weighed, the correction is of great importance, and must be computed with great care. The value of σ depends on the temperature of the air in the balance case and on the barometric pressure, both of which should always be recorded when an exact weighing is made. It also depends to a less degree on the humidity of the atmosphere, and the effect of this may usually be neglected. If the balance case is kept dry by sulphuric acid or some other drying agent, this factor is largely eliminated. Table V., Appendix, contains values of the weight of one cubic centimeter of air under various conditions.

In applying the above correction, it is most convenient to increase or diminish the apparent weight w by a certain per cent as determined by the value of the correction term compared with unity (see formula). Thus, if a mass of glass weighs 5.1131 grams, its weight in vacuo will be 1.00034 times greater, i.e., 0.034 per cent greater:

1 per cent of $5.1131 = 0.051$; 0.034 per cent $= 0.034 \times 0.051 = 0.0017$

which gives at once the correction to be added. Note that two significant figures are amply sufficient in this case for computing the correction term, whereas if 5.1131 × 1.00034 is multiplied out nothing is gained in accuracy, and much labor is wasted.

THE BALANCE.—PART I.

METHODS OF WEIGHING.

Before beginning this experiment, study carefully the preceding General Discussion and the Theory of the Balance. The manipulation of the balance and procedure to be followed in weighing will be explained and illustrated by an instructor. Careless handling of a delicate balance by an inexperienced person is liable to result in serious injury to the balance.

Object.—The object of this experiment, is *first*, to give practice in the use of an analytical balance by weighing a substance both by the Ordinary or Equal Swing Method and by the Method of Swings, and, *second*, to compare the volume of the substance computed from its weight and density with the volume calculated from its linear dimensions, the substance being a true geometrical figure.

Apparatus.—The apparatus provided is a good equal arm balance sensitive to 0.1 milligram and provided with a rider. The substance to be weighed is a carefully turned right cylinder. Vernier calipers reading to 0.02 mm. are also provided for measuring the dimensions of the cylinder.

Procedure.—*First.*—Take the balance as it stands, level it by means of the spirit level or plumb bob, and determine the weight of the substance by the "Method of Swings," described on p. 35.

Second.—Adjust the balance for weighing by the "Ordinary Method," p. 35, and weigh the substance again.

Third.—Record the temperature and barometric pressure in the laboratory as given by the self-recording instruments.

Fourth.—Measure the linear dimensions (diameter and length) of the cylinder with vernier or micrometer calipers taking four independent measurements of each.

Computation.—*First.*—The density of the substance weighed will be given by an instructor. Reduce the apparent weight of the substance to its true weight in vacuo, taking as the apparent weight the mean of the two determinations made. The value of σ (the density of the air) is to be taken from Table V., Appendix.

Second.—Compute the volume of the substance in cubic centimeters from its true weight and compare with the value calculated from its dimensions.

Precision Discussion.—The volume of a cylinder is $V = \frac{1}{4}\pi d^2 h$ where d is the diameter and h is the height. Suppose the precision measures of h and d are δ_h and δ_d respectively. These may be assumed equal to the *A.D.*'s of the measurements. What is the precision measure of the final result, *V?*

First Solution.—The deviation in V due to the deviation δ_h in h alone will be

$$\begin{aligned}\Delta_h &= \frac{\partial}{\partial h}\left(\tfrac{1}{4}\pi d^2 h\right).\ \delta_h \\ &= \tfrac{1}{4}\pi d^2\, \delta_h\,.\end{aligned}$$

Similarly the deviation in V due to the deviation δ_d in d alone will be

$$\begin{aligned}\Delta_d &= \frac{\partial}{\partial d}\left(\tfrac{1}{4}\pi d^2 h\right).\ \delta_d. \\ &= \tfrac{1}{4}\pi h.\ 2d.\ \delta_d\,.\end{aligned}$$

The resultant deviation in V due to the combined effect of both deviations will be

$$\begin{aligned}\Delta &= \sqrt{\Delta_h{}^2 + \Delta_d{}^2} \\ &= \sqrt{(\tfrac{1}{4}\pi d^2.\ \delta_h)^2 + (\tfrac{1}{4}\pi h.\, 2d.\ \delta_d)^2}.\end{aligned}$$

Second Solution.—(Always simpler when the function is a product, quotient or power of the measured quantities).

The fractional deviation of d is $\dfrac{\delta_d}{d}$.

" " " " h is $\dfrac{\delta_h}{h}$.

Therefore the fractional deviation in V due to the fractional deviation in h is, by inspection, $\dfrac{\Delta_h}{V} = \dfrac{\delta_h}{h}$ (as h enters as first power in a product).

Similarly the fractional deviation in V due to the fractional deviation $\frac{\delta_d}{d}$ in d is $\frac{\Delta_d}{V} = 2\,\frac{\delta_d}{d}$ (as d enters as a square).

The resultant fractional deviation in V is, therefore,

$$\frac{\Delta}{V} = \sqrt{\left(\frac{\Delta_h}{V}\right)^2 + \left(\frac{\Delta_d}{V}\right)^2},$$

$$= \sqrt{\left(\frac{\delta_h}{h}\right)^2 + \left(2\,\frac{\delta_d}{d}\right)^2},$$

and hence

$$\Delta = V\sqrt{\left(\frac{\delta_h}{h}\right)^2 + \left(2\,\frac{\delta_d}{d}\right)^2}.$$

This method evidently involves very much less work than the preceding.

Questions and Problems.—1. Which method of determining the volume do you consider the more reliable, and why? (The error in the value of the density given may be considered not greater than one part in 5000.)

Optional Problems.

2. From the *A.D.* of the diameter measurements (δ_d), and the *A.D.* of the length measurements (δ_h), find the percentage deviation of d, h, and of the final result V. What is the corresponding actual deviation of the volume in cubic centimeters?

3. Is it necessary to reduce the weight to vacuo in computing the volume, if the result is desired to only 0.1 per cent? to 0.05 per cent?

THE BALANCE.—PART II.

CURVE OF SENSITIVENESS OF A BALANCE AND RATIO OF BALANCE ARMS.

Before performing this experiment, read carefully pp. 27–34 on *General Theory of the Balance.* Students must also have performed "*The Balance, Part I,*" or already have had practice in the use of a delicate balance.

Object.—The object of this experiment is two-fold: *first,* to determine the sensitiveness of a balance under various loads; and *second,* to determine the ratio of its arms.

Apparatus.—The apparatus provided is a delicate, analytical balance sensitive to 0.1 milligram.

Procedure.—*Preliminary adjustments.*—*First,* level the balance by means of the spirit level or plumb bob attached to the balance. *Second,* determine the period of vibration of the beam by noting the time of several transits of the pointer through the lowest part of its arc. With a short arm balance the time of a single vibration should not be over eight or ten seconds; with a long arm balance it should not exceed twelve or fourteen seconds. In case these limits are surpassed, speak to an instructor, who will adjust the centre of gravity of the beam to a proper distance below the knife edge.

Sensitiveness.—The sensitiveness of a balance has been defined to be the angle through which the beam rotates under an excess of load of one milligram in either pan. As this angle is proportional to the number of scale divisions over which the pointer moves, the latter is usually taken as a measure of the sensitiveness instead of the angle itself.

The sensitiveness is to be determined with loads of 0, 10, 20, 30, 40 and 50 grams in each pan. The "Method of Swings" is to be employed throughout. The procedure is to determine the point of rest as described on page 35; *first,* with say, zero grams in each pan, and *second,* with one milligram weight added with the rider on the right side of the balance beam. The difference of the points of rest under these two conditions is the number of divisions through which the pointer is deflected by an excess of weight of one milligram. Proceed in the same way to determine the sensitiveness for each of the above loads.

Ratio of Arms.—The ratio of the length of the balance arms could be obtained most simply with a set of weights which were exactly adjusted among themselves, or at least to as high a degree of precision as it is desired to ascertain the ratio of the balance arms. With such a set of weights it would simply be necessary to find the weight w_r in the right-hand pan, which would balance a weight w_l in the left-hand pan; then, if r and l are the lengths of the right and left arm of the balance, respectively,

$$w_r r = w_l l$$

$$\frac{r}{l} = \frac{w_l}{w_r}.$$

Weights cannot be assumed to be adjusted to the degree of precision required for this work without a special comparison (see p. 45), and hence the ratio of the arms of a balance is best determined as follows: Weigh some substance of weight a, first in the left-hand pan, and then in the right-hand pan. Suppose w_r and w_l are the weights (very nearly the same) required to balance it in these two cases, respectively. Then

$$al = w_r r$$

$$w_l l = ar$$

Eliminating a we have

$$\frac{r^2}{l^2} = \frac{w_l}{w_r},$$

or

$$\frac{r}{l} = \sqrt{\frac{w_l}{w_r}} = \sqrt{1 + \frac{w_l - w_r}{w_r}}$$

$$= 1 + \frac{w_l - w_r}{2w}, \text{ approximately.}$$

Note that in this case the same weights may be used for w_l and w_r, the slight difference between them being added or subtracted by means of the rider. The errors of the set of weights are thus eliminated. As only two or, at most, three significant figures are retained in $\frac{w_l - w_r}{2w}$, w may be taken as either w_l or w_r.

Determine in this way the ratio of the arms of the balance using for a a weight about one-half the maximum load for which the balance is designed.

Computation.—Plot a curve of sensitiveness of the balance with values of the sensitiveness as ordinates and the corresponding loads as abscissæ. In choosing scales, be careful that the scale of ordinates does not exceed ten times the precision of the data, otherwise the plot will be much distorted, and it will be found difficult to draw a "best representative line."

Questions and Problems.—1. What conclusion can be drawn from the curve of sensitiveness regarding the construction of the balance beam? (Compare p. 33.)

2. To what precision may the balance be used without correcting for the ratio of its arms? What is the minimum weight which would be affected to one milligram by this correction? To one-tenth milligram?

THE BALANCE.—PART III.

CALIBRATION OF A SET OF WEIGHTS.

Object.—To determine the errors of a set of weights.

A set of weights furnished by the best makers cannot as a rule be relied upon closer than to one milligram, and hence for exact work in absolute weighing it is necessary to determine not only the relative errors of the weights among themselves, but also the absolute error of the set by comparison with a *standard*. By the calibration of a set of weights is meant the determination of the ratio of each single weight to the true combined weight of all together, as determined by a comparison with a standard.

Apparatus.—The apparatus required is a delicate balance sensitive to 0.1 or 0.05 milligram, a set of weights and a standard weight.

Procedure.—The procedure is as follows: All weighings are to be made by the "Method of Swings" (p. 35). It will be assumed that the balance arms are equal to within the precision which it is desired to calibrate the weights. If this is not the case, the ratio of the arms must be first determined as described in Part II, and each weighing corrected by this amount.

Suppose the set of weights is made up as follows: 50, 20, $10_{(1)}$, $10_{(2)}$, 5, $2_{(1)}$, $2_{(2)}$, 1. The fractional platinum weights will not be considered. The subscripts (1) (2) denote different weights of the same denomination. The weights are then to be compared in the following order:

$$
\begin{array}{rcl}
\textit{Left-hand pan.} & & \textit{Right-hand pan.} \\
5+2_{(1)}+2_{(2)}+1 & = & 10_{(1)}+a \\
10_{(2)} & = & 10_{(1)}+b \\
20 & = & 10_{(1)}+10_{(2)}+c \\
50 & = & 20+10_{(1)}+10_{(2)}+5+2_{(1)}+2_{(2)}+1+d
\end{array}
$$

a, *b*, *c*, *d* being the small weights which it is found necessary to add to or subtract from the right-hand side of the balance in order to establish equilibrium. Suppose further that the combined weight w of all the weights compared with a standard 100 gram weight is found to be

$$w = 50 + 20 + 10_{(1)} + \ldots + 1 = 100 + e$$

where e may, of course, be plus or minus.

If each weight is now expressed in terms of some one weight, as $10_{(1)}$ taken for reference, we have

$$
\begin{array}{rcl}
5 + 2_{(1)} + 2_{(2)} + 1 & = & 10_{(1)} + a \\
10_{(1)} & = & 10_{(1)} \\
10_{(2)} & = & 10_{(1)} + b \\
20 & = & 2 \times 10_{(1)} + b + c \\
50 & = & 5 \times 10_{(1)} + a + 2b + c + d
\end{array}
$$

and hence adding these equations,

$$w = 10 \times 10_{(1)} + 2a + 4b + 2c + d = 100 + e$$

This may be written,

$$10\,[10_{(1)} + \tfrac{1}{10}\,(2a + 4b + 2c + d - e)] = 100$$

or

$$10_{(1)} + \tfrac{1}{10}(2a + 4b + 2c + d - e) = \tfrac{100}{10} = 10 \text{ grms. (standard).}$$

Hence if
$$\delta = \tfrac{1}{10}\,(2a + 4b + 2c + d - e),$$
$$10_{(1)} = 10 - \delta.$$

The true value of each of the weights in terms of the standard will therefore be

$$\begin{aligned}
5+2_{(1)}+2_{(2)}+1 &= 10-\delta+a\\
10_{(1)} &= 10-\delta\\
10_{(2)} &= 10-\delta+b\\
20 &= 20-2\delta+b+c\\
50 &= 50-5\delta+a+2b+c+d
\end{aligned}$$

from which the numerical correction to be applied to each weight is readily determined.

The correction for the 5, $2_{(1)}$, $2_{(2)}$, and 1 gram weights and fractional weights can, of course, be determined in like manner. If the weights are to be used for relative weighings only, as in most chemical work, the comparison with the standard may be omitted and the value of e in the above formulæ taken as 0.

SPECIFIC GRAVITY DETERMINATIONS.

GENERAL DISCUSSION.

The *density* of a body is the mass or quantity of matter contained in the unit of volume. The numerical value of the density depends, therefore, on the system of units in which the mass is measured. Thus in the English system of weights and measures the density of water is 62.5, the mass of one cubic foot (unit of volume) being 62.5 pounds. In the centimeter-gram-second (c.g.s.) system, the only system in scientific use, the density of water is unity, since the unit of mass, the gram (the one-thousandth part of the mass of the standard kilogram), is practically equal to the mass of one cubic centimeter of water.

The *specific gravity* of a substance is the ratio of the weight of a given volume of the substance to the weight of an equal volume of water at 4° C., the temperature of its maximum density. Specific gravity is, therefore, a pure number independent of the system of units employed. From the above relation between the units of mass in the c.g.s. system and the mass of one cubic centimeter of water, it follows that the numerical value of the density of a body in the c.g.s. system is identical with its specific gravity. This is not absolutely true however, as it has been found that the mass of one cubic centimeter of water at 4° C. is not exactly equal to one gram. The difference is so slight that for most purposes it may be neglected.

The specific gravity of a substance is sometimes erroneously referred to water at some other temperature than 4° C. This is frequently done with liquids which are compared with water at the same temperature as the liquids themselves. Thus sp. gr. benzene 0.808 $^{20°}/_{20°}$ means that the weight of equal volumes of benzene and water, both at 20°, are in the ratio 0.808 to 1. The true specific gravity of benzene at 20°, as defined above, is somewhat less, and should be expressed thus: 0.808 $^{20°}/_{4°}$. The temperature, both of the substance and of the water to which it is referred, should therefore always be stated to avoid ambiguity.

The effect of changes of barometric pressure on the specific gravity of liquids and solids is negligible. This is not true, however, in the case of gases and vapors. Unless otherwise specifically stated, these are always reduced to "normal conditions," i.e., to 0° C., and 760 mm. pressure. Moreover, the standard substance to which they are referred is dry air or hydrogen, under the same conditions of temperature and pressure, instead of water. The specific gravity of a vapor, as distinct from a gas, is often spoken of as its "vapor density," although no reason other than the custom warrants the use of the expression.

The *specific volume* of a substance is the volume occupied by a unit mass of it under standard conditions of temperature and pressure or, in other words, the reciprocal of its density. In the c.g.s. system it may be defined as the volume occupied by one gram of the substance under standard conditions.

The *molecular* and *atomic volume* of a substance, quantities of frequent use and much importance in chemistry, are the volumes occupied by one molecular mass and one atomic mass of the substance, respectively. They are obtained by multiplying the specific volume by the molecular and atomic mass (weight), of the substance, respectively.

The various methods in common use for determining specific gravity all depend on determining, first, the weight of the substance itself; and second, the weight of an equal volume of the standard substance water (or air). The principal methods in use for determining the specific gravity of solids and liquids are described below. For gases and vapors *Ostwald's Physico-Chemical Measurements*, pp. 100–106, may be consulted.

SPECIFIC GRAVITY OF SOLIDS.—I.

Object.—To determine to 0.1 per cent the specific gravity of a solid insoluble in water, with a specific gravity flask. The computation illustrates the application of temperature corrections and reduction of weighings to vacuo.

Discussion.—In this very convenient and accurate method, the weight of a volume of water equal to the volume of the substance is obtained by determining the weight of water displaced from a small flask or bottle, when the substance is introduced. This is done by weighing the flask, first, when completely filled with water or with the water adjusted to some reference mark; and second, when the flask contains both substance and water, the total volume being again adjusted the same as before. From these two weights, together with that of the substance dry, the weight of the water displaced by the immersed solid can be at once determined. The accuracy of the determination depends on the precision with which the contents of the flask can be adjusted successively to the same volume.

In Fig. 14 are shown three of the best forms of specific gravity bottles in general use. All are provided with carefully ground stoppers. The stopper of *A* has a capillary bore and the water is adjusted so as to completely fill the flask and stopper. In *B* and *C* the liquid is adjusted to a reference mark etched on the narrow stem. *C* is provided in addition with a sensitive thermometer, ground to fit the neck of the flask, for indicating the temperature of the liquid.

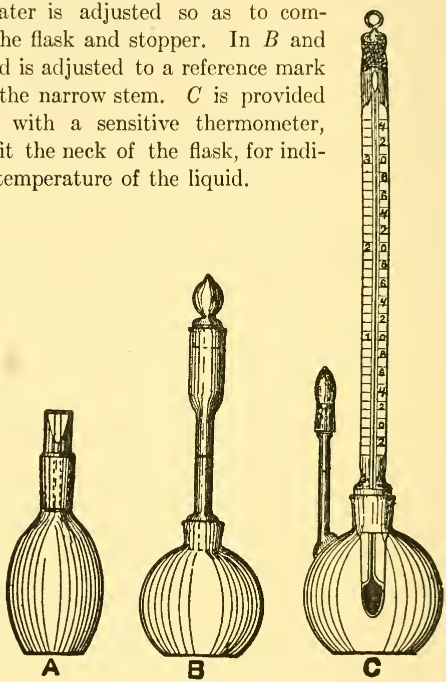

Fig. 14.

Apparatus.—The apparatus provided is some form of specific gravity flask illustrated above and a good analytical balance.

Procedure.—Weigh the substance dry. Clean the specific gravity flask with bichromate cleaning solution if necessary, rinse with distilled water and then with alcohol, and dry by allowing a current of warm air to blow into the bottle.[1] Obtain the weight of the bottle to 0.01 gram (for reasons given below). Next fill the bottle with distilled water, making sure that no air bubbles adhere to the sides, and carefully insert the glass stopper firmly but not tightly, without enclosing any air. If the glass be thin, special care must be taken not to force the stopper in and so distort and change volume of the flask. Adjust the liquid by means of filter paper to the reference mark if either flask B or C is used. Dry the outside carefully, particularly around the neck of the stopper, and weigh. Immediately afterwards take the temperature t_1 of the water, by inserting the bulb of the thermometer into the bottle. Next drop the substance into the flask, refilling with more water if necessary, and replace stopper as before. See that no air bubbles are caught between the pieces of substance introduced. Again weigh, and record the temperature t_2 of the water. Record also the barometer height and the temperature of the air in the balance case.

---

[1] A special arrangement for drying bottles is provided at the sink.

If the flask with its contents is adjusted both times at the same temperature by immersing it in a constant temperature water bath, the corrections for expansion of glass and change of density of the water given below may be eliminated.

Computation.—The specific gravity of the substance at $t_2°$ referred to water at 4° C. is to be computed, all corrections being applied which affect the final result to 0.1 per cent.

Let $w =$ apparent weight of substance dry;
$w_1 =$ " " " bottle plus water at $t_1°$;
$w_2 =$ " " " " water and substance at $t_2°$;
$b =$ " " " bottle dry.

The approximate specific gravity of the substance is

$$s = \frac{w}{w_1 + w - w_2}. \tag{1}$$

The value of s thus obtained is likely to be in error by a per cent or more, due to the omission of the following corrections:

First, for change of volume of the bottle due to the change of temperature from $t_1°$ to $t_2°$;

Second, for change in density of the water due to the same temperature change;

Third, for buoyancy of the air.

The first two corrections may often be made negligible, as pointed out above, by making $t_1°$ and $t_2°$ sufficiently near the same. The third correction must always be applied unless shown to be smaller than the experimental error; this is often the case when the specific gravity of the substance is nearly unity. The above three corrections are made as follows:—

Correction for change of temperature.—We wish to find what the weight w_1 would have been, had the bottle plus water been weighed at $t_2°$ instead of $t_1°$. The weight of water filling the flask at $t_1°$ is $w_1 - b$. The volume of the flask at $t_1°$ is $v_{t_1} = \frac{w_1 - b}{D_{t_1}}$, where D_{t_1} is the density of water at $t_1°$. The volume of the flask at $t_2°$ is, therefore,

$$v_{t_2} = v_{t_1} [1 + k\ (t_2 - t_1)],$$

where $k = 0.000026$ is the mean coefficient of cubical expansion of the glass between $t_1°$ and $t_2°$. Hence the weight of water of density D_{t_2} filling this volume at $t_2°$ will be

$$\begin{aligned} v_{t_2} \times D_{t_2} &= v_{t_1} [1 + k\ (t_2 - t_1)]\ D_{t_2}° \\ &= (w_1 - b)\ [1 + k\ (t_2 - t_1)]\ \frac{D_{t_2}}{D_{t_1}}. \end{aligned}$$

Hence the weight of the bottle plus water at $t_2°$ would be

$$\begin{aligned} w_2' &= b + (w_1 - b)\ [1 + k\ (t_2 - t_1)]\ \frac{D_{t_2}}{D_{t_1}} \\ &= b + (w_1 - b)\ [1 + k\ (t_2 - t_1) + D_{t_2} - D_{t_1}] \text{ approx.} \\ &= w_1 + (w_1 - b)\ [k\ (t_2 - t_1) + D_{t_2} - D_{t_1}]. \qquad (2) \end{aligned}$$

The weight of the water displaced by the solid is, therefore,

$$w' = w_2' + w - w_2. \qquad (3)$$

Discussion.—It is to be noticed that the value of the second term in formula (2) is a small correction to be added to or subtracted from the observed weight w_1. If w_1 is weighed to 0.001 gram, the correction is negligible if less than 0.0005 gram; if to 0.0001 gram, it should be computed to the nearest 0.1 milligram. In general two significant figures in each of the factors are sufficient, as the whole correction is seldom greater than a few milligrams. It is for this reason that the weight of the bottle b need only be known approximately. To illustrate the magnitude of the correction involved, suppose the capacity of the bottle is about 30 cc., that $t_1° = 20°.0$ and $t_2° = 21°.0$ C. Then $w_1 - b = 30$ approximately, and b need only be weighed to 0.1 gram. The value,

$D_{t_2} - D_{t_1} = 0.99804 - 0.99825 = -0.00021$ (see Appendix, Table VIII).

Hence the correction per degree at 20° to be applied to w_1 is

$$30 \times [0.000026 - 0.00021] = -0.0055 \text{ grams.} \qquad (4)$$

The correction due to change in density of the water is seen to be considerably greater and in the opposite direction to that due to the expansion of the glass. For a bottle of even 10 cc. capacity the above correction is not negligible for a variation of one degree, if the weighings are carried to milligrams. For a larger bottle the correction would be proportionately greater.

Reduction to vacuo.—If W and W' be the true weights in vacuo of the solid and of the displaced water respectively (see p. 38),

$$W - \frac{W}{s}\sigma = w - \frac{w}{\Delta}\sigma,$$

$$W' - \frac{W'}{D_{t_2}}\sigma = w' - \frac{w'}{\Delta}\sigma.$$

Dividing one equation by the other, we obtain

$$\frac{W\left(1 - \frac{\sigma}{s}\right)}{W'\left(1 - \frac{\sigma}{D_{t_2}}\right)} = \frac{w\left(1 - \frac{\sigma}{\Delta}\right)}{w'\left(1 - \frac{\sigma}{\Delta}\right)},$$

$$\frac{W}{W'} = \frac{w}{w'}\,\frac{\left(1 - \frac{\sigma}{D_{t_2}}\right)}{\left(1 - \frac{\sigma}{s}\right)} = \frac{w}{w'} \cdot \frac{(D_{t_2} - \sigma)}{(s - \sigma)} \cdot \frac{s}{D_{t_2}},$$

but $\frac{W'}{D_{t_2}} = \frac{W}{s} =$ true volume of substance

$$\text{or} \quad \frac{W}{W'} = \frac{s}{D_{t_2}}.$$

$$\text{Hence} \qquad s = \frac{w}{w'}\,(D_{t_2} - \sigma) + \sigma. \tag{5}$$

The value of s thus computed is the value of the specific gravity of the substance at the temperature $t_2°$ referred to water at 4° C. If it is further desired to compute the specific gravity of the substance at another temperature, say $t°$, the mean coefficient of cubical expansion of the substance must be known. If this be β, then

$$s_{t°} = s_{t°_2}\,[1 + \beta\,(t_2° - t)],$$

since the specific gravity is inversely proportional to the volume of a given mass. The exact equation for this reduction is

$$s_t = s_{t_2}\,\frac{1 + \beta_1 t}{1 + \beta_2 t_2}$$

where β_1 and β_2 are the mean coefficients of expansion between, say 0° and $t°$ and 0° and $t_2°$ respectively. Practically, however, $\beta_1 = \beta_2$ for most substances, but unless β be known with sufficient accuracy, a greater error may be introduced in the resulting value of s than occurs in the actual experimental work.

Problem.—1. With a flask of 25 c.c. capacity, what is the greatest allowable difference which the temperatures t_1 and t_2 may have and the correction for expansion of the glass and density of water in formula (2) be negligible, *first*, if the weighings are assumed accurate to one milligram, and, *second*, to one-tenth milligram? [Solve approximately: see equation (4).]

SPECIFIC GRAVITY OF SOLIDS.—II.

Object.—To determine the specific gravity of a solid substance by the application of Archimedes' Principle.

Discussion.—If a substance is weighed first, in air, and, second, when suspended in a liquid in which it is insoluble, the loss in weight is, by Archimedes' Principle, equal to the weight of the liquid displaced by the substance. If the liquid be water, the weight of a volume of water equal to that of the substance is thus determined. Water is always chosen as the liquid unless the substance is soluble in it, in which case benzene, or some other organic liquid is used. The method may evidently be employed to determine the specific gravity of the liquid instead of that of the solid, if the specific gravity of the latter is known.

Apparatus.—The apparatus required is a delicate analytical balance provided with a table for supporting a beaker of water over one pan, some very fine wire or fibre for suspending the substance, and a thermometer.

Procedure.—Find the weight w of the substance dry. Next suspend it by a very fine wire of such a length that the substance will be completely immersed in a beaker of water which sets upon a little table placed over the balance pan. Let the weight of the substance when thus freely suspended in distilled water be w_1. Owing to the great damping effect of the water, the weighings must be made by noting the position where the pointer of the balance comes to rest, and not by the usual method of equal swings. The method of swings may be employed when the damping is not too great. Record the temperature of the water immediately after weighing. Finally remove the solid from the wire supporting it and weigh the latter with its lower end immersed in the water to the same extent as during the preceding weighing. Let the weight of the wire weighed in this manner be w_2. Then $w' = w + w_2 - w_1$ will be the weight of the water displaced by the substance alone, the error due to immersed wire being thus eliminated. If, on the other hand, the wire be weighed dry and not partially immersed, a correction for the buoyant effect of the water on that portion immersed when the substance is suspended must be made, or shown to be negligible. This correction may usually be made negligible by using a sufficiently fine suspension. Record the barometer reading, and the temperature of air in balance case.

The above method is capable of a high degree of accuracy when all details are properly attended to. The suspending fibre, or wire, should be as fine as possible in order to reduce to a minimum the error arising from capillarity at the point where the fibre leaves the surface of the liquid. A cocoon fibre has been found, when practicable, to give excellent results. The substance must hang freely in the liquid and not approach too near the sides of the beaker. If several pieces are suspended together, great care must be taken to prevent air bubbles from remaining enclosed between them. Air bubbles may usually be removed by placing the beaker and contents under the receiver of an air pump and exhausting.

Computation.—The specific gravity of the substance at t° referred to water at 4° C. is, approximately,

$$s = \frac{w}{w'} D_t ,$$

when D_t is the density of water at t°. The only correction to be applied to the above expression is that for the reduction of the weights w and w' to vacuo. For this reduction, see formula (1), page 38.

SPECIFIC GRAVITY OF LIQUIDS.—I.

Object.—To determine the specific gravity of a liquid to 0.1 per cent by means of a specific gravity bottle.

Apparatus.—Flasks of the form shown in Fig. 14, p. 50, may be used for this purpose; when intended for liquids the necks of the flasks are usually made considerably narrower than when intended for solids.

Procedure.—Clean thoroughly, dry and weigh the flask empty. Fill completely with the liquid whose specific gravity is to be determined, or adjust the liquid by means of filter paper exactly to the reference mark. Weigh, and immediately after record the temperature t_2 of the liquid. Thoroughly rinse the flask with distilled water, and then fill with distilled water previously boiled to remove dissolved air. Adjust again, weigh, and record the temperature t_1 of the water; or adjust the flask with water at the same temperature t_2 as before, in order to eliminate temperature corrections.

Computation.—Compute the specific gravity of the liquid at $t_2°$ referred to water at 4°, applying all corrections affecting the result to more than 0.1 per cent.

Let $b =$ weight of flask empty;
$w_1 =$ " " " filled with water at $t_1°$;
$w_2 =$ " " " " " liquid at $t_2°$;
$D_{t_1} =$ density of water at $t_1°$;
$\sigma =$ " " air in balance case at time of weighing.

The approximate specific gravity of the liquid is

$$s = \frac{w_2 - b}{w_1 - b}.$$

This value is, however, in error for the same causes as those discussed under specific gravity of solids, p. 52.

Correction for temperature.—The weight of water which would have filled the flask at $t_2°$ is (see p. 52),

$$w' = (w_1 - b)\,[1 + k\,(t_2 - t_1)]\,\frac{D_{t_2}}{D_{t_1}}.$$

Hence $$s = \frac{w_2 - b}{w'},$$

referred to water at $t_2°$,

and $$s = \frac{w_2 - b}{w_1 - b} \cdot \frac{D_{t_1}}{1 + k\,(t_2 - t_1)},$$

referred to water at 4° C.

Reduction to vacuo.—If both the weight of the liquid and of the water are further reduced to vacuo, we have (see p. 54)

$$s = \frac{w_2 - b}{w'} \cdot (D_{t_2} - \sigma) + \sigma.$$

This last reduction is usually unnecessary; for, writing the equation in the form,

$$s = \frac{w_2 - b}{w'} \cdot D_{t_2} + \sigma\left(1 - \frac{w_2 - b}{w'}\right),$$

it is seen that, if $s = \frac{w_2 - b}{w'} = 2$, the value of the correction term $\sigma\left(1 - \frac{w_2 - b}{w'}\right)$ is only -0.0012; hence neglecting the correction entirely will not introduce an error greater than one part in 2000, or 0.05 per cent.

SPECIFIC GRAVITY OF LIQUIDS.—II.

Object.—To determine the specific gravity of a liquid to 0.1 per cent by means of a Sprengel-Ostwald picnometer.

Apparatus.—A much more convenient form of apparatus for accurate specific gravity determinations of liquids is the Sprengel picnometer as modified by Ostwald and shown in Fig. 15. It is constructed so that when filled with water or with the liquid the adjustment may be readily made at the same temperature by suspending the whole instrument, except its capillary ends, in a bath kept at constant temperature. All temperature corrections are thus eliminated. The capacity varies from 10 cc. to 50 cc., according to the accuracy of the results desired. The picnometer is most readily filled (or emptied) by sucking (or blowing) through a small rubber tube connected to the end *A*.

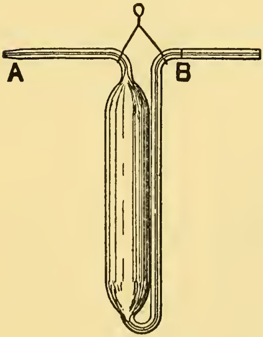

Fig. 15.

Procedure.—Thoroughly clean the picnometer with a solution of potassium bichromate in sulphuric acid if necessary, rinse with distilled water, and finally with clean alcohol. Warm in a drying closet or by a drying blast, and dry by drawing through it a current of air.

When the picnometer has cooled to the room temperature, weigh it. Next fill with the liquid whose specific gravity is to be determined, and suspend the picnometer up to the horizontal ends in a water bath, the temperature of which is carefully adjusted, by the addition of hot or cold water, to the temperature at which the gravity determination is to be made, which is 20° C. The filling is most readily accomplished by drawing in the liquid at *B* by sucking through a small rubber tube connected to the end *A*, as shown in Fig. 16.

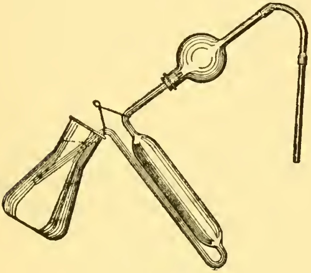

Fig. 16.

After about five minutes, bring the meniscus of the liquid to the reference mark *B*, by carefully touching the point *A* with a bit of filter paper. If the meniscus then remains perfectly stationary at the mark *B*, the picnometer and its contents have assumed the temperature of the surrounding bath. If not, the adjustment should be repeated. A little liquid may be added without removing the picnometer from the bath by touching a drop on the end of a stirring rod to the capillary end *A*.

When the adjustment has been made, dry the picnometer, and weigh it. In case the room temperature is much above that of the bath, precautions must be taken to prevent the liquid from expanding beyond the ends of the picnometer while weighing. This may be prevented by cooling the liquid (not below the dew-point however), before weighing. To reduce the time of weighing to a minimum, it is advantageous to have the approximate weight on the balance pan beforehand. For very expansive and volatile liquids, small ground glass cups are sometimes placed over the ends of the picnometer to prevent loss during weighing.

In the same way as described above, find the weight of the picnometer filled with distilled water, rinsing the picnometer thoroughly three or four times before filling.

Computation.—Since the weight of equal volumes of liquid and water are here compared at the same temperature, the corrections for expansion of glass and change of density of water are eliminated. If w is the weight of the liquid, and w_1 and D_t the weight and density of the water respectively at $t°$, the corrected specific gravity of the liquid at $t°$ referred to water at 4° is (see equation (4), page 54).

$$s = \frac{w}{w_1}(D_t - \sigma) + \sigma.$$

It will readily be seen that the reduction to vacuo is here negligible if the specific gravity of the liquid does not vary much from unity. The specific gravity of the liquid at 20° C. is to be referred both to water at 20° C. and to 4° C.

Problems.—1. If the coefficient of expansion of glass is 0.000026 per degree and the density of water diminishes 0.00021 per degree at 20°, how closely should the temperature of the water bath be adjusted in filling the picnometer in order that the combined error due to expansion of glass and change of density of the water may not be greater than 0.1 milligram. Assume volume of the picnometer to be 10 cubic centimeters.

2. Is it necessary in general to reduce weighings to vacuo in determining the specific gravity of liquids? Why?

THE MOHR-WESTPHAL BALANCE.

Object.—This experiment gives practice in the manipulation of a Mohr-Westphal balance, by means of which the specific gravity of liquids may be determined with great ease and rapidity to about 0.1 or 0.2 per cent.

Apparatus.—The principle of this balance is as follows: At the end of a balance arm, which is divided into ten parts, is suspended a thermometer in the form of a glass sinker. When suspended freely in air, this is exactly balanced by a fixed counterpoise at the other end of the balance beam. In the usual type of balance the volume of the sinker is such that when completely immersed in distilled water at 15° C., it displaces five grams of water. Hence, if immersed in distilled water, it will require the addition of a weight A hung from the end of the beam, equal to the weight of this volume of water, in order to bring the balance again to equilibrium. The large weights A are adjusted therefore to equal the weight of water displaced by the sinker at the standard temperature chosen,— 15° C. A series of weights B, C, and D equal respectively to $\frac{1}{10} A$, $\frac{1}{100} A$, and $\frac{1}{1000} A$ are also provided. If, therefore, the sinker is suspended in any other liquid than water whose specific gravity is to be determined, and it is found that, in order to bring the balance to equilibrium, the weights A, B, C, D have to be placed on the beam at position a, b, c, d respectively, these being expressed in tenths of the length of the beam, the specific gravity will be given at once as $0.abcd$. For,

$$s = \frac{\text{weight of displaced liquid}}{\text{weight of equal volume of water}},$$

$$= \frac{\frac{a}{10}A + \frac{b}{10}B + \frac{c}{10}C + \frac{d}{10}D}{A},$$

$$= \frac{\frac{a}{10}A + \frac{b}{10}\frac{A}{10} + \frac{c}{10}\frac{A}{100} + \frac{d}{10}\frac{A}{1000}}{A},$$

$$= 0.abcd.$$

Procedure.—Determine the specific gravity of the same liquid which was used with the Sprengel-Ostwald picnometer. Adjust the balance by turning the screw at the base so that the sinker is exactly balanced in air. Suspend the sinker in distilled water and see that the weight A equals the weight of water displaced at 15°. Then measure the specific gravity of the solution provided, at 20° C.

Computation.—Refer the specific gravity of liquid at 20° to water at 4° C. and compare result with that obtained with the picnometer.

THE JOLLY BALANCE.

Object.—This experiment affords practice in the manipulation of a Jolly Spring Balance, an experimental investigation of Hooke's Law, and illustrates a ready method of determining specific gravities without the use of an equal arm balance or set of weights.

Discussion.—By Hooke's Law the elongation of a spring is directly proportional to the force applied; or if a coiled spring be suspended at one end and weighted at the other, the elongation of the spring will be proportional to the weight applied. This relation is true as long as the elastic limit of the spring is not approached. If the elongation corresponding to unit weight, one gram, is determined once for all, a measurement of the elongation produced by any substance suspended by the spring affords all necessary data for deducing its weight. The elongation per unit weight, k, is a constant for a given spring except in so far as the elasticity of the spring varies with the temperature. The constant depends, of course, on the dimensions and elasticity of each individual spring, and may be varied within very wide limits. In certain instruments springs of extraordinary sensitiveness are often employed.

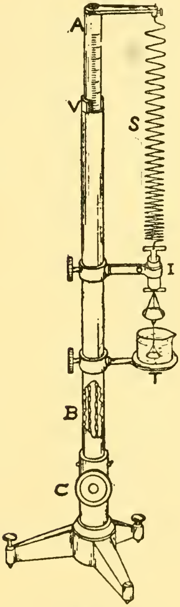

Fig. 17.

If the ratio of two weights, w_1 and w_2, is desired, as in specific gravity determinations, and a_1 and a_2 are the respective elongations of the spring which they produce, we have

$$\frac{w_1}{w_2} = \frac{ka_1}{ka_2} = \frac{a_1}{a_2};$$

hence, whenever relative measurements only are involved, the value of the constant k need not be known.

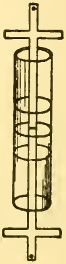

Fig. 18.

Apparatus.—The form of apparatus provided is shown in Fig. 17. A spring, *S*, is suspended from the top of a tube, *A*, which can be raised or lowered by means of an endless chain enclosed within the tube, *B*, and operated by the screw, *C*. The tube *A* is graduated in millimeters, and its height is accurately determined to 0.1 millimeter by means of a fixed vernier, *V*. From the lower end of the spring is suspended a light aluminum index *I*, and a double pan. The pans and their suspending wires are made as light as possible. The upper is made of aluminum, and the lower, which is immersed in a beaker of water, is of glass or mica. The index is suspended, as shown in Fig. 18, within a glass tube, on which are etched three reference lines. A reference mark is also scratched on the index itself. The tube carrying the index is adjustable in height as well as the support *T*, for carrying the beaker.

Procedure.—Lower the column *A*, by the screw *C*, as far as possible so that all weight is removed from the spring. Clamp the support *T*, at the lower end of *B*, and adjust *I* at such a height that the lower pan is suspended near the bottom of a beaker of distilled water placed on *T*. Raise the spring gradually until the index *I* is raised off the top of the glass support, and then level the whole apparatus by the screws at the base until the index is freely suspended at the centre of the glass cylinder. The apparatus is then ready for use.

If the spring oscillates excessively, it can be quickly brought to rest by lowering it until the index just rests on the glass support, and then very gradually raising it again until the index swings freely. The water, in which the lower pan should always be immersed, also acts as a very effective damping medium to the oscillations of the freely suspended spring.

First.—To study the sensitiveness of the apparatus and to test Hooke's Law.

Raise the spring until the reference mark on the index is exactly at the height of the middle circle etched on the glass cylinder. As these reference marks are etched completely around the cylinder, the error due to parallax is easily avoided by bringing the eye to such a height that the mark on the index coincides with the plane of the etched circle. Record the reading of the vernier V. Make four independent settings and take the mean m_0 as the setting for zero load. Next place a one-gram weight in the upper pan, and raise A until the index again comes to the same reference mark as before, and read the vernier. Make four independent settings and take the mean m_1. The difference $m_1 - m_0$ is the elongation of the spring for one gram. Determine the elongation with two, three, and four grams in the upper pan, making four settings with each.

Second.—To determine the specific gravity of a solid substance.

See that the beaker is filled with fresh distilled water. Adjust the spring so that the index is exactly at the reference mark with no load in either pan, and call the vernier reading a_0. Place the substance whose specific gravity is to be determined in the upper pan and adjust the index again to the reference mark, calling the reading a_1. Finally place the substance in the lower pan so that it is completely immersed in the water and call the reading of the vernier when the index is again brought to the reference mark a_2. Be careful that no air bubbles adhere to the substance when immersed. The temperature, $t°$, of the water should be taken immediately after the experiment. Make a duplicate determination.

The specific gravity of the substance will evidently be given by the expression,

$$s = \frac{a_1 - a_0}{a_1 - a_2} D_t,$$

where D_t is the density of the water at $t°$.

Third.—To determine the specific gravity of a liquid.

Replace the beaker of water by a beaker containing the liquid to be investigated, and determine the difference in elongation $a'_1 - a'_2$ of the spring, when the same substance previously used is placed in the upper and lower pan respectively. Record the temperature $t°_1$ of the liquid. The difference $a'_1 - a'_2$ is proportional to the loss of weight of the substance in the given liquid, and hence, dividing this by the elongation proportional to the loss of weight of the same substance in water, $a_1 - a_2$, we obtain the specific gravity of the liquid at $t°_1$ referred to water at $t°$. If the specific gravity of the solid substance is known, a single determination of the elongation of the spring when the substance is weighed first in air and second when immersed in the given liquid, gives of course, all data necessary for computing the specific gravity of the latter.

Computation.—*a. To ascertain whether the apparatus follows Hooke's Law within the experimental error involved in the manipulation.*—Compute the mean and *a.d.* of each series of four settings with 0, 1, 2, 3, and 4 grams load. Find the elongations $k_0 = m_1 - m_0$; $k_1 = m_2 - m_1$, etc., per gram for the several loads and compute their mean and *a.d.* This last *a.d.* gives the amount by which the elongation of the spring per gram is likely to differ from the mean value of the elongations for loads from one to four grams. It depends both on the error in setting the instrument and on the deviations of the elongation of the spring from strict proportionality to the weights applied under different loads, i.e., deviations from Hooke's Law.

To determine the error due to setting alone we may compute the indeterminate error of each value of k and take their mean, or proceed as follows:

Take the arithmetical mean of all deviations of setting, i.e.,

$$\frac{a.d._0 + a.d._1 \ldots + a.d._4}{5} = \overline{a.d.}$$

This gives a fair estimate, from twenty settings under various conditions, of the accuracy with which the experimenter can adjust the index to the reference mark. Hence the average deviation of any elongation computed from the difference of two such settings will lie between the extreme values $2\,\overline{a.d.}$ and 0 according as the deviations in the two settings happen to be of the same or opposite sign. Since, however, the sign of accidental deviations is never known and is equally likely to be plus or minus, the most probable value of the deviation in the elongation will be

$$\Delta = \sqrt{\overline{a.d.}^2 + \overline{a.d.}^2} = \overline{a.d.}\sqrt{2}.$$

(See Precision of Measurements, p. 29.)

Compare this result with the average deviation of the elongations k_0, k_1, etc., and state whether the instrument follows Hooke's Law within the experimental error or not. Another method of investigating this point would be to plot values of k as ordinates and corresponding total loads as abscissæ and note whether the points deviate regularly or not from a line parallel to the axis of X.

b. Compute the specific gravity of the solid, referring it to water at 4° C.

c. Compute the specific gravity of the liquid at $t°_1$, referring it to water at 4° C.

BAROMETRIC MEASUREMENTS.

GENERAL DISCUSSION.

One of the quantities, the value of which is very frequently required in physical, chemical, and engineering work, is the pressure of the atmosphere at the time and place at which an observation is made. This quantity is measured by some form of barometer, and is usually spoken of as the barometric pressure. A number of different forms of barometer are in common use, the most important of which may be classed as either mercurial or aneroid barometers. Primary instruments are of the former type; the latter are always secondary, i.e., they must be calibrated by a comparison with a standard instrument. Both types include forms of self-registering instruments.

By the barometric height is meant the height of a column of pure mercury at 0° C. which just balances the pressure of the atmosphere at the time and place of the observation. In order that measurements taken by different instruments and at different places may be comparable, they must be reduced to standard conditions. This involves the application to the observed height of several corrections, which will be discussed below.

The *standard* or *normal barometric pressure* is defined as the pressure of a column of pure mercury 76 cm. high at 0° C. As this pressure varies with the latitude and altitude owing to the variation in gravity, it is necessary to further stipulate in what latitude and at what altitude the pressure shall be considered standard. Latitude 45° and sea level have been so chosen, under which conditions $g = 980.6\ ^{\text{cm.}}/_{\text{sec.}^2}$ The normal barometric pressure may therefore be expressed in any of the following ways: the pressure per square centimeter of a column of pure mercury 76 cm. high at 0° C., latitude 45° and sea level, or the pressure per square centimeter of $76 \times 13.596 = 1033$ grams weight in latitude 45° and sea level, or the pressure per square centimeter of $1033 \times 980.6 = 1013200$ dynes. This pressure is also called the *normal atmosphere*, and is very frequently used as the unit of pressure in work on the mechanics of gases.

Corrections.—The corrections which must be applied to the observed height of a barometer in order to reduce it to standard conditions are the following:

First.—Correction for temperature of mercury and scale.

Let h = observed height of barometer at $t°$ C. If the scale on which the height is measured be standard at $t°$, or if it be of a material the coefficient of expansion of which is so small that the changes in its length from the standard are negligible for ordinary temperature changes, then the reduction of the barometric height to 0° C. involves simply the computation of the contraction of the mercury column in cooling from $t°$ to 0°. This may be calculated from the formula,

$$h_t = h_o (1 + a t),$$

from which

$$h_o = h_t \frac{1}{1 + a t},$$

$$= h_t (1 - a t) \text{ approximately,} \qquad (1)$$

$$= h_t - a t h_t,$$

where a is the mean coefficient of expansion of mercury between 0° and $t°$ C. The value of a from Regnault's experiments as reduced by Broch is $a = 0.0001818$ between 0° and 100° C. The correction is therefore, $0.0001818\ t\ h_t$ and will be subtractive above 0° and additive below this temperature. For ordinary laboratory conditions, say $t = 20°$ and $h = 760$ mm., the value is about 2.76 mm. If the barometer is read only to the nearest 0.1 mm., it is usually sufficient to use the formula

$$0.0001818 \times 760\, t = 0.138\, t,$$

which gives the correction to two significant figures with sufficient accuracy for pressures in the neighborhood of 760 mm.

If however, as is usually the case, the value of the scale unit is not standard at the temperature of the measurement, the above value h_o must be further corrected for the expansion of the scale.

Let β = the linear coefficient of expansion of the material of which the scale is constructed;
t = observed temperature of the scale at time of measurement (assumed the same as temperature of the mercury);
t' = temperature at which scale is standard;
h_o' = the value of h_o reduced to standard units of length. Then

$$h_o' = h_o[1 + \beta\,(t - t')].$$

If the scale is standard at 0°, this becomes

$$h_o' = h_o\,(1 + \beta t).$$

Combining this with (1) we have

$$\begin{aligned} h_o' &= h_t\,[1 - (a - \beta)\,t] \text{ approximately.} \\ &= h_t - (a - \beta)\,th_t, \end{aligned} \qquad (2)$$

an expression in which the effect of the expansion of the scale and of the mercury are combined into one correction term. The value of β varies with the nature of the material of which the scale is constructed. The values for some materials in common use are given below:

Brass	0.0000184	to	0.000019
Glass	0.0000085	to	0.0000088
Steel	0.0000245		
Pine wood along grain	0.0000037		
" " across "	0.000058		

The scales on most barometers are of brass. For such barometers the correction for any observed height h_t and temperature t, is therefore given by the expression $(0.0001818 – 0.0000184)\ th_t$, or $0.0001634\ th_t$.

The value of this correction computed for temperatures ranging from 0° C. to 35.8° C. and for pressures from 440 to 795 millimeters of mercury may be found in the Smithsonian Meteorological Tables, pp. 34–56. Table IX, appendix, is taken from this source. For any temperature below 0°, the value of the correction will evidently be numerically equal but of opposite sign to that for the same interval above 0°. For barometers reading in English units, the mercury column is reduced to its height at 32° F. and the scale is referred to its length at 62° F., the temperature at which the standard yard is standard. The corrections for barometers with brass scales have been computed for every 0.5° F. from 0° F. to 100° F. and for pressures varying from 19 to 31.6 inches of mercury, and will be found in Table 10 of the Smithsonian Tables and in Table X of the appendix.

Second.—Correction for Capillarity.

The readings of all cistern barometers in which the cross section of the barometer tube is less than a certain amount, require a correction for the depression of the mercury column, due to capillarity. The magnitude of the depression depends on the internal diameter of the barometer tube and on the angle which the mercury surface makes with the wall of the tube. As the depression is by no means constant for a given tube, but varies with the second factor mentioned above, the height of the meniscus must be known in every case as well as the diameter of the tube. The correction is an uncertain one at best. Values for the correction, interpolated from observations of Mendelejeff and Gutkowsky, are given in Table XIV. For tubes 20 mm. in diameter the correction amounts to only 0.025 mm., and for tubes 25 mm. or more in diameter the correction becomes entirely negligible. It is therefore theoretically much better to construct barometers with large tubes at least 25 mm. in diameter, and thus eliminate this source of error, instead of attempting to correct for it. Barometers so constructed are called *normal barometers.* Such barometers are not very portable owing to the liability of breakage arising from the large quantity of mercury contained in the tube.

Third.—Correction for the tension of mercury vapor.

This correction is very small except at high temperatures. It is negligible when an accuracy not greater than 0.1 mm. in the barometric pressure is desired. Values of the correction at various temperatures are given in Table XIII, appendix.

Fourth.—Correction for variation of gravity with the latitude.

The pressure of a given column of mercury at any place is proportional to the force of gravity obtaining at that place. As stated above, latitude 45° and sea level are the conditions chosen as standard. The value of g in latitude 45° and at sea level is

$$g_{45} = 980.6 \frac{\text{cm}}{\text{sec}^2}.$$

In any other latitude, ϕ, at sea level,

$$g = g_{45}(1 - 0.00259 \cos 2\,\phi).$$

The reduced height of the barometer must therefore be multiplied by the ratio $\frac{g}{g_{45}}$ to reduce it to latitude 45°.

Fifth.—Correction for variation of gravity with the altitude.

The variation of the value of g with the altitude H above sea level is given, for moderate altitudes, by the expression

$$g' = g(1 - \frac{2H}{R}) = g(1 - 0.00000020\,H),$$

where R the radius of the earth and H are expressed in meters. The last two corrections for variation of gravity with latitude and altitude may be combined into one formula as follows:

$$g' = g_{45}(1 - 0.0026 \cos 2\,\phi - 0.0000002\,H).$$

Types of Barometers.—For the procedure to be followed in filling a barometer, see Wüllner "Lehrbuch der Experimentalphysik," I, p. 403. For description of numerous types of this instrument, see Deschanel's "Natural Philosophy," Part I, Chap. xvii; Ganot's "Physics," pp. 151–160. Only the types used in the laboratory are here described.

A barometer in its simplest form consists of a glass tube about 85 cm. long closed at one end, which after being completely filled with pure mercury is inverted in a trough of mercury as shown in *A*, Fig. 19. The mercury sinks in the tube to such a height that the weight of the column of mercury above the level of the mercury in the trough is exactly equal to the weight of the atmosphere. Since the difference in height between the top of the mercury column and the surface of the mercury in the trough varies from day to day, with the variations in atmospheric pressure, some device is necessary for always measuring the height from the level of the surface in the cistern. A convenient arrangement for this purpose is a steel rod *C* of known length which can be moved parallel with the barometer tube by a rack and pinion and set so that its lower end just makes contact with the surface of the mercury in the cistern. The height of the barometer under these conditions may then be found by adding the length of the rod to the difference in height of the top of the rod and the top of the meniscus, as read on a vertical scale. These readings may be conveniently and accurately made with a reading telescope. A cathetometer may also be used for measuring the height. *B*, Fig. 19, represents a second form of barometer tube enlarged at the top in order to diminish or eliminate the effect of capillarity.

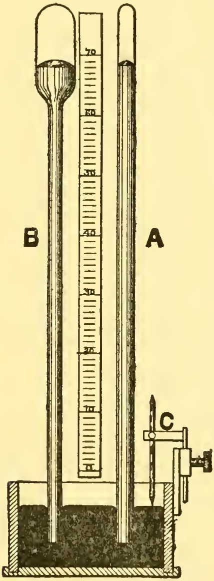

Fig. 19.

The Fortin Barometer.—This well-known and very practical form of barometer is shown in Fig. 20. The barometer tube, which tapers to a small opening at its lower end, projects well into a cistern, the construction of which is the characteristic feature of the instrument. It is usually made of a cylinder of boxwood, to the top of which is fixed a cylinder of glass to permit the mercury surface being seen. The lower end of the cistern is made adjustable in height by being closed by a flexible bottom of leather, or leather lined with pure rubber. The adjustment is made by means of a screw S, which presses against a bearing attached to the bottom of the leather pouch. The glass cylinder is closed at the top by a cover through which passes the barometer tube, as shown in the figure. The connection between the tube and cover is made by chamois leather, which permits the air to pass freely between the cistern and the outside atmosphere, but which at the same time prevents mercury from leaking out when the cistern is full. The cover is of metal, but lined with boxwood, or other non-metallic coating to prevent contact of the mercury with the metal.

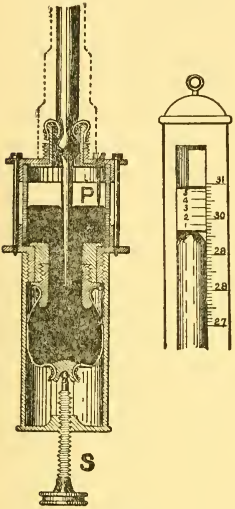

Fig. 20.

From the top of the cover an ivory point, P, projects downward, the position of the lower end of which is taken as the zero point for adjusting the scale on which the height of the barometer is read. This scale is attached to the upper part of the tube, as shown in Fig. 20, and is provided with a vernier which is set by means of a rack and pinion. The whole barometer is encased in metal, except at the top and bottom, where it is cut away to permit the mercury surfaces being seen. It is provided with a thermometer for indicating the temperature of the mercury and scale, the bulb being placed inside the metal case next to the barometer tube.

To set the barometer, the screw, *S*, at the bottom of the instrument is first turned down until the surface of the mercury in the cistern is well below the end of the reference point, *P*. The mercury is then raised slowly until the end of the ivory point just coincides with its image reflected in the surface of the mercury or until it makes a minute dimple in the mercury surface. The latter method is, in general, more accurate. This procedure always insures a rising meniscus in the barometer tube. The height of the barometer is then read by first raising the vernier above the mercury meniscus and then gradually bringing it down until the lower edge is in a plane exactly tangent with the top of the meniscus. This is accomplished by the aid of a second plate similar to the vernier and moving with it at the back of the barometer tube. The lower edge of this plate is in a plane with the lower edge of the vernier and perpendicular to the tube. If the eye be brought just in line with these two edges, the line of sight will be at right angles to the tube; hence, in setting the vernier on the meniscus, the eye should be kept in such a position that both edges simultaneously appear to be tangent to the highest point of the meniscus. This adjustment is very important for the correct reading of the instrument.

The two chief advantages of this type of instrument are the ease and accuracy with which the surface of the mercury in the cistern may be adjusted to the same point, and its portability. The proportions are such that when the bottom of the cistern is screwed up the mercury will fill not only the cistern, but also the barometer tube. Under these conditions the barometer may be transported in any position without danger of air entering the tube.

Kew Barometer.—This is another form of cistern barometer, in which the area of the cross-section of the barometer tube bears a known ratio to that of the cistern. The scale is then so graduated that the divisions are shortened by an amount which just compensates for the change in the level of the mercury in the cistern, thus making unnecessary the adjustment of the mercury in the cistern to a reference point before each reading as in barometers of the Fortin type. One reading of the position of the mercury column gives the barometric height at once. The instrument though very convenient is less reliable than the Fortin.

Siphon Barometer.—The siphon barometer in its simplest form is shown in *A*, Fig. 21. It consists of a bent tube 85 to 90 cm. long, the longer end of which is closed, and the shorter open to the atmosphere. When completely filled with mercury and mounted in the position shown, the column of mercury in the closed arm is just balanced by the weight of mercury in the open arm plus the pressure of the atmosphere. To bring the meniscus in the upper arm vertically over that in the lower, the tube is usually bent as shown in *B*.

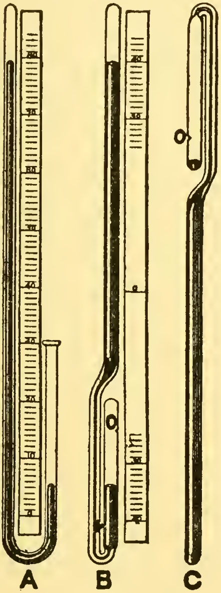

Fig. 21.

The height of the mercury column may be read either by means of a cathetometer or by means of a telescope and scale. In some instruments the scale is made adjustable so that its zero may be brought to the level of the mercury in the open arm; in others the scale is fixed, and the whole barometer is adjustable in the same way to the scale; in both of these cases the reading of the position of the upper meniscus gives the desired height. More frequently however, the barometer is read by two verniers of similar construction to those described above. In this case it is usual to graduate the scale only near the ends of the barometer. The zero of the scale is at the centre of the tube and the graduations numbered up and down from this point so that the difference in height of the mercury in the open and closed arms is given by the *sum* of the two vernier readings. These correspond to the position of the lower edge of each vernier when set tangent to the meniscus in each arm of the barometer respectively.

One of the obvious advantages of this type of barometer is the elimination of the correction for capillary depression, for if the diameter of the two arms where the meniscus stands is the same, the depression should be the same in each. To ensure this however, it is necessary to tap the barometer tube sharply before each reading.

In order to make barometers of this type portable, various devices have been introduced into their construction.[1] One form, due to Gay-Lussac, is shown in *B*, Fig. 21. The two arms are connected by a capillary tube bent as shown in the figure. The short arm is sealed except for a capillary hole, *O*. To transport the barometer without admitting air into the longer arm, the barometer is tilted until the mercury completely fills the long arm, when it is inverted as in *C*, the excess of mercury falling to the bottom of the short arm under the opening, *O*. It cannot escape through this minute opening owing to capillarity. The longer arm, being completely filled, can suffer no jar from a motion of the mercury, neither can air easily enter it through the capillary tube.

Standard Barometer.—One excellent form of standard barometer, constructed on the siphon principle, is shown in Fig. 22. The barometer tube proper *A*, at least 25 mm. in diameter, is drawn out to a tapering end which reaches nearly to the bottom of a steel cistern provided with two

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[1] See Wüllner's Lehrbuch der Experimentalphysik, I, p. 412.

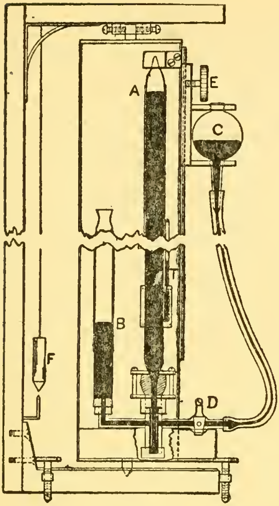

Fig. 22.

lateral arms. The tube is held air tight in this cistern by means of a rubber stopper, which is clamped between two brass plates the lower of which forms the top of the cistern. One arm of the cistern is provided with a steel stopcock, *D*, which in turn is connected to a reservoir of mercury, *C*, by means of heavy pressure tubing. This reservoir can be raised or lowered and clamped at any desired height. The other arm of the steel cistern ends in a right angle elbow, and communicates with the open arm, *B*, of the barometer which is carefully chosen of the same cross-section as the tube, *A*.

The whole instrument is mounted on a rigid board pivoted above and below. The mounting itself is rendered vertical by means of a plumb bob, *F*, and levelling screws in the base. Slots several inches long are cut in the board behind the mercury meniscus in *A* and *B* respectively, and adjustable mirrors are provided for illuminating the meniscus from behind. A fine thermometer, *T*, is mounted along the barometer column to record the temperature of the mercury.

Provision is made for mounting a standard meter scale between *A* and *B* in the plane of the mercury columns if it is desired to read the barometric height by means of a reading telescope instead of a cathetometer.

To adjust this barometer, first level the instrument by means of the levelling screws in the base. Clamp the reservoir a little higher than the top of the meniscus in *B*, and carefully open the cock, *D*. The mercury in *B* will rise slowly. With the cock still open lower *C* until the mercury in *B* begins to fall, and continue to allow the mercury to flow back into *C* until the mercury in *A* stands about ten centimeters below the top of the barometer, *A*. Close the stop-cock. Now raise *C* and clamp it at such a height that the mercury will rise in both arms *A* and *B* when the stop-cock is opened. To set the barometer, open the stop-cock and allow the mercury to slowly rise a few millimeters in *A* and *B*. Then close it. This procedure insures a rising meniscus in each arm of the barometer, and if both are of equal cross-section the error due to capillarity becomes eliminated. By choosing the cross-section of the tubes, 25 mm. or more in diameter, this error is rendered practically negligible in any case. The height of the barometer may then be read either by means of a cathetometer, or a telescope and scale. The latter method is to be preferred. A second and third setting should be made by opening *D* and allowing the mercury to take up a new position in both arms.

Aneroid Barometers.—Barometers of this type are constructed without the use of any liquid (whence their name). Numerous forms are in common use, but all are based on the same principle. One form, known as the Richards recording barometer or barograph, is shown in Fig. 23. The essential part of the instrument consists of a series of flattened cylindrical boxes of thin metal placed on top of each other and separated by small pieces of metal attached to the centre of each. The boxes are corrugated so as to render them

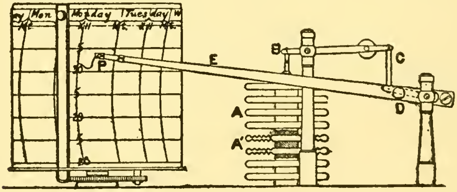

Fig. 23.

more flexible and yielding to change of pressure. Each box is partially exhausted and hermetically sealed. An increase in the external pressure therefore tends to flatten each element. By connecting them all in a series as shown, the displacements due to this deformation are added and transmitted by means of a system of multiplying levers, *B*, *C*, *D*, to a long index arm, *E*. This is made to move over a scale graduated to give the barometric pressure at once in inches or millimeters; or a pen, *P*, is attached to its end by means of which a record of the barometric changes is traced on a revolving drum as shown in the figure. The drum revolves by clockwork, making one revolution a week.

Aneroid barometers may be made exceedingly sensitive to slight changes of pressure; — a variation in the pressure of the atmosphere for a change in altitude of only a few feet may be detected. They may also be made portable and no larger than an ordinary watch. Such pocket barometers are very convenient for approximate altitude determinations. The chief disadvantages of the instrument arise from the effect of changes of temperature on its indications and from its liability to get out of order. Frequent comparisons with a standard barometer are necessary if its indications are to be relied upon.

BAROMETERS.

Before performing this experiment, the general discussion of barometric measurements, pp. 67–79, should be carefully studied.

Object.—The object of this experiment is to familiarize the student with the construction and manipulation of various types of barometers, and with the exact reduction of barometric pressures to standard conditions. The principles involved in this reduction are of fundamental importance in all pressure measurements. Practice in the use of standard meteorological tables, and the computation of altitudes, is also afforded.

Apparatus.—The barometers to be studied are several forms of simple cistern barometers, the Fortin barometer, the Kew barometer, the siphon barometer, the standard barometer, and the aneroid barometer.

Procedure.—Study the construction of each barometer and its method of use. Set and read each barometer at least twice, recording the temperature of the attached thermometer in each case. The standard barometer is not to be read (simply inspected), unless the instructor so directs.

Computation.—Reduce the mean reading of each barometer to standard conditions applying all corrections which affect the result to 0.1 mm. Data on the internal diameter of the several barometer tubes will be given by an instructor. English barometers are to be reduced to metric units for comparison with the metric barometers. Tabulate all results.

Problems.—1. Deduce the formula for the reduced height of an English barometer at $t° = 32°$ F., the scale being of brass and standard at 62° F. Show why the correction is subtractive above 28°.5 F. and additive below this temperature (see p. 70).

2. As an exercise in the use of barometric tables for computing altitudes, the student is recommended to solve the following problem, complete directions for which will be found in the Smithsonian Meteorological Tables, 1896, pp. xxix to xxxi. Given:—

Barometric pressure reduced to 0° C. at upper station = 635.8 mm.

Barometric pressure " " " " lower " = 767.3 mm.

Mean temperature of air column between stations = 14°.3 C.

Mean vapor pressure = 8 mm.

Latitude = 44°.

Lower station is 27 meters above sea level.

(*a*) Find the altitude of upper station.

(*b*) Compute the altitude also by Babinet's formula, (p. xxxii, Smithsonian Tables), and compare results.

METEOROLOGICAL INSTRUMENTS.

Object.—The object of this exercise is to familiarize the student with the principle, construction and use of the more common instruments employed in meteorological observations, i.e., in determining the pressure, temperature and humidity of the atmosphere, the velocity of the wind, etc. The reduction of the observations gives practice in the use of standard Meteorological Tables.

Apparatus.—The collection of instruments includes the following:—

First.—Barometers and barographs. See preceding experiment on Barometric Measurements.

Second.—Thermometers.

1. Six's maximum and minimum thermometer.
2. Rutherford's maximum thermometer, U. S. Weather Bureau type.
3. Rutherford's minimum thermometer, U. S. Weather Bureau type.
4. Richard's recording thermograph.

Third.—Hygrometric apparatus.

1. Negretti and Zambra wet and dry bulb thermometer or psychrometer.
2. Lambrecht polymeter.
3. Hygrodeik.

Fourth.—Wind recording apparatus.

1. Cup anemometer for measuring velocity of wind.
2. Vane anemometer for measuring velocity of draughts in flues.

Procedure.—*First.*—Record the reading of the Fortin barometer and its attached thermometer, as these data are necessary in later computations. A study of the various types of barometers is to be taken as a separate exercise.

Second.—Inspect the several self-registering thermometers and see that the principle of their construction is thoroughly understood. A description will be found in the reference books placed with the apparatus.

Third.—Determination of the hygrometric conditions of the atmosphere.

This includes the determination of (*a*) the *pressure* of the water vapor present in the atmosphere, (*b*) the *dew point*, (*c*) the *humidity* or *relative humidity*, and (*d*) the *weight of vapor* in the unit of volume.

The quantity of water present in the atmosphere may be determined in a number of ways, the most direct and accurate of which is to pass a known volume of air through drying tubes and weighing the amount of water absorbed in the tubes. This is the fundamental method by which are obtained the data on which psychrometric tables are based. As this procedure is very tedious other simpler though less direct methods are usually employed. Of these the most reliable is that based on the indications of a wet and dry bulb thermometer.

The principle of this instrument is based upon the fact that water at the temperature of the air evaporates less rapidly the greater the amount of water vapor present; also that, in evaporating, the water absorbs heat from bodies with which it is in contact. Hence the drier the air the more rapid will be the evaporation at the bulb of the wet thermometer, and consequently the greater the difference in reading between it and the dry bulb thermometer. In absolutely saturated air both thermometers will read the same. The simplest relation which has been found to hold for the difference in temperature of a wet and dry bulb thermometer and the pressure of the vapor in the air is

$$f = f_1 - AB\,(t - t_1),$$

in which

t = temperature of the air (dry bulb);
t_1 = temperature of wet bulb thermometer;
f = pressure of aqueous vapor in the air at $t°$;
f_1 = pressure of saturated vapor at $t°$;
B = barometric pressure;
A = a constant.

The value of A depends to a certain extent on the form and size of the wet bulb thermometer and velocity of ventilation.

It has been found, however, that if the air in the neighborhood of the wet bulb moves at a rate of about ten feet or more per second, a constant maximum depression is reached, and the effect of the form of instrument is eliminated. Observations should therefore be made in a current of air whose velocity is such that a maximum depression is obtained, or the thermometer should be swung so as to give it a corresponding ventilation, as is done in the U. S. Weather Bureau observations.

In the apparatus as arranged, ventilation is secured by means of a small fan driven by an electric motor. Read the two thermometers under this condition taking care not to warm either during the observation.

From these observations the following data expressing the condition of the atmosphere are to be looked out in the hygrometrical tables of the Smithsonian Collection (see pp. xxxv to xlii, last edition, 1896). These tables are computed from direct measurements of the humidity of the air and the corresponding differences in readings of a wet and dry bulb thermometer, taken under the conditions described above.

1st. *Temperature.*—Given by dry bulb thermometer.

2d. *Vapor Pressure.*—To find the pressure f of aqueous vapor turn to Table 40, p. 134, and with t_1 as argument find f_1, the pressure of saturated vapor at the temperature of the wet bulb thermometer. This gives the first term in the formula for f. The value of the second term is found in Table 41, by taking the barometric pressure B, in inches, and the observed temperature difference $t - t_1$ as arguments.

$$f = \text{table } 40 - \text{table } 41.$$

3d. *Dew-point.*—The "dew-point," i.e., the temperature at which the aqueous vapor in the atmosphere would begin to be precipitated, or in other words, the temperature at which the actual vapor present would be just sufficient to saturate the air, is evidently the value of t in Table 40 corresponding to the value of f just found.

4th. The *relative humidity,* or simply *humidity,* of the air is the ratio of the quantity of vapor actually present to that which would be present if the air were saturated at the temperature of observation; i.e., relative humidity $= \frac{f}{F}$, where F is the value for saturated vapor at $t°$, the temperature of the dry thermometer. The value is to be found in Table 40. Humidity is usually expressed in percentages. Thus, relative humidity = 75 per cent, or 0.75 denotes that the air contains three-fourths of the amount of moisture required to saturate it at the temperature of observation. This ratio, $\frac{f}{F}$, may also be defined as the ratio of the maximum vapor pressure at the dew-point d, to that at the observed temperature t, and corresponding to this definition, values of $\frac{f}{F}$ are given in Table 42 for arguments $t - d$ (horizontal column) and t (vertical column). Corresponding tables are also given for metric units and centigrade degrees.

5th. *Weight of Vapor.*—The amount of moisture in the air is also sometimes expressed as the actual weight of water which the air contains as vapor in the unit of volume; e.g., as grains per cubic foot or grams per cubic meter. To find the weight of vapor present, turn to Table 38, which contains the weight of saturated vapor in air at different temperatures, and look out the value corresponding to the dew-point d, since at this temperature the vapor present would just saturate the air.

The hygrodeik is an empirically graduated instrument based on the indications of a wet and dry bulb thermometer. The hygrometric data are plotted on a chart in such a way that if an index is adjusted to the reading of the wet bulb thermometer the corresponding humidity, dew-point, etc., may be read off the chart at once. The readings of this instrument (Lloyd's) will be found to agree very well with the results interpolated from the tables. It is a very convenient instrument for ordinary work.

The polymeter (Lambrecht's) is another secondary instrument for indicating hygrometric data, i.e., relative humidity, dew-point and vapor pressure. It is based on the principle of the hair hygrometer. The indications of this instrument are, under ordinary conditions, much less reliable than those of the preceding instruments. For details of construction and method of reading the instrument, see descriptive pamphlet with the apparatus.

The readings of all three instruments are to be recorded and results tabulated for comparison.

Anemometers.—These instruments are designed for measuring the velocity of the wind or of air currents in general.

The "cup anemometer" used by the U. S. Signal Service, is that most commonly used for measuring wind velocities. It consists of four arms at right angles to each other at the ends of which are attached hemispherical cups. A current of air striking these cups meets with less resistance from the convex than from the concave side, and hence causes a rotation in the direction of least resistance. The number of rotations is recorded by a suitable mechanism. Wind velocities are usually expressed in miles per hour.

The vane anemometer is a much more delicate instrument and is adapted for measurements of such air currents as occur in the ventilating system of buildings. It consists of a very delicately pivoted wheel carrying a number of inclined aluminum vanes; when placed in a current of air the vanes revolve with a velocity depending on the velocity of the air current. The wheel transmits its motion to a series of dials which record the number of revolutions. The recording mechanism may be thrown in or out of gear at will, while the vanes are revolving, by a small lever on the side of the instrument. These instruments require careful calibration in a current of air of known velocity. The corrections to be applied vary greatly with the velocity owing to friction of the bearings, and the accurate calibration of one of these instruments is not a simple matter. For small velocities an approximate correction can be obtained by walking with the anemometer at a constant rate for a known distance, a hundred feet or more, in a room where the air is at rest, and noting the time.

The instruments provided are to be inspected carefully. The velocity of the incoming or outgoing air in one of the flues of the laboratory is to be measured with the vane anemometer.

BOYLE'S LAW.—I.

Object.—This experiment is designed to illustrate the relation between the pressure and volume of a gas, dry air, at room temperature, for pressure less than one atmosphere. The precision aimed at is about 0.3 per cent. The computation illustrates the graphical method of deducing the empirical law connecting two quantities, a series of values for which has been determined by experiment.

Apparatus.—The apparatus consists of a heavy walled glass cistern *A*, nearly filled with mercury, into which a glass tube *B*, sealed at the top, can be raised or lowered at will. This tube is about one meter in length and is graduated from the top downwards in centimeters and millimeters. The exact volume in cubic centimeters corresponding to any division is determined once for all by calibrating the tube with known volumes of mercury before the apparatus is set up. The corrections thus obtained, which are to be added to the linear readings in order to convert them into the corresponding volumes in cubic centimeters, are given on the calibration curve provided with the apparatus. The tube *B* is initially completely filled with pure mercury and inverted in the cistern. The apparatus then constitutes a simple barometer, the height of the mercury column in *B* above the level of the mercury in the cistern being just balanced by the atmospheric pressure. In the apparatus as set up a little dry air has been allowed to enter the vacuum above the mercury column, so that when the tube *B* is raised to the stop *C*, the air is under a pressure of about half an atmosphere. The tube is held in any

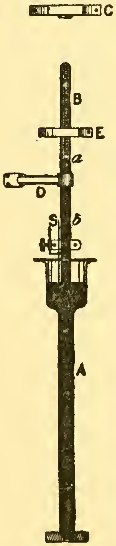

Fig. 24.

desired position by a clamp *D*. To facilitate reading the height of the mercury column above the variable height of the surface of the mercury in the cistern, a steel rod, *S*, is provided, which may be raised or lowered parallel with the tube *B*, by a rack and pinion. The cistern is covered with a glass plate to prevent dust settling on the mercury surface.

Procedure.—Before beginning the experiment proper, record the height of one of the barometers (the siphon), and also the reading of the thermometer attached to the apparatus. Raise the tube *B* up to the stop *C*, and read the position of the top of the mercury column. In setting the tube for this and all subsequent measurements, the precaution should always be taken to tap the tube or to set with a rising meniscus so as to insure that the meniscus is convex. The reading is best taken by clasping a strip of black paper around the manometer and bringing it down until its lower edge is in a plane tangent to the top of the meniscus. Next adjust the steel rod, *S*, so that the lower end coincides with its image in the mercury of the cistern. Record the position of the upper end of the rod *S* on the manometer. Care must be taken in making this observation to bring the eye exactly on a level with the end of the pointer, otherwise a serious error due to parallax will be introduced. This error may be avoided if the eye be moved until the top of the pointer appears to coincide with its own reflection in the mercury in the manometer. Repeat these observations at seven or eight different heights of the tube, taking them over as wide a range as possible. In setting the tube great care should be taken not to warm the upper portion containing the air, by the hand or otherwise. The tube is best handled with a handkerchief at the lower portion containing the mercury, as a slight change in temperature of this produces little effect on the resulting pressure or volume of the gas. The success of the experiment depends on maintaining the gas at as nearly a constant temperature throughout all measurements as possible. Sudden changes of volume are also to be avoided, as they produce adiabatic heating or cooling

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[1] For Method of Calibration, see Stewart and Gee's Practical Physics, vol. i, p. 109.

of the gas. Finally remove the pointer S and measure its length on a steel scale. The barometer and temperature of the surroundings should also be again recorded.

Computation.—The pressure on the gas at any position of the tube is evidently equal to the atmospheric pressure expressed in centimeters of mercury, diminished by the height of the mercury column in the tube B above the level of the mercury in the cistern, expressed in the same units. This is easily computed from the observed data. The corresponding volume occupied by the gas is obtained by adding to the observed reading, the proper correction interpolated from the calibration plot accompanying the apparatus.

Tabulate the values of the pressure, p, expressed in centimeters of mercury and of the corresponding volumes, v, in cubic centimeters; in a third column tabulate also the values of $\frac{1}{v}$, for reasons explained below. (Look out these values in a table of reciprocals, or use a slide rule.) To determine the form of the function between the quantities p and v, supposing it unknown, first, make a direct plot with values of p as ordinates, and v as abscissæ. The points will be found to lie along a curved line which by inspection, from its symmetry with respect to the axes, might be suspected of being an hyperbola referred to its asymptotes as axes. The equation of such a curve is $xy =$ constant, or $y = c\,(\frac{1}{x})$. This equation may be transformed into the equation of a straight line by changing the variable x to $x' = \frac{1}{x}$. Making this substitution we obtain $y = cx'$, the equation of a straight line passing through the origin and making an angle whose tangent is c, with the axis of x. If, therefore, a second plot be constructed on the same sheet with the same values of $p = y$ as ordinates, and with the reciprocals of the volumes $\frac{1}{v} = \frac{1}{x} = x'$ as abscissæ, the points should lie along a straight line passing through the origin if the gas follows Boyle's Law.

This will be found to be the case if the data have been accurately taken. *Draw the best representative line, and determine its equation.* Follow directions for reciprocal functions discussed under Graphical Methods, p. 50.

The relation may also be tested by computing the product pv for each pair of values of p and v and discussing the results analytically or graphically. Plotting the values of pv as ordinates and values of p as abscissæ, the line should be parallel to the axis of abscissæ for a perfect gas.

Precision Discussion.—What would be the percentage change produced in the product pv by a change of temperature of 1° C.?

$$\begin{aligned} pv &= p_o v_o\,(1 + a\,t), \\ &= \frac{p_o v_o}{273}\,(273 + t), \\ &= RT. \end{aligned}$$

Let $$x = pv = RT;$$

then $$\frac{\Delta}{x} = \frac{\delta_T}{T},$$

or the fractional change in x is equal to the fractional change in the absolute temperature. If the temperature at the time of the experiment is 18° C., $T = 273 + 18 = 291°$, and $\delta_T = 1°$, therefore,

$$\frac{\Delta}{x} = \frac{1}{291},$$

or $$100\,\frac{\Delta}{x} = \frac{100}{291} = 0.34 \text{ per cent.}$$

Hence a change in temperature of 1° at 18° will introduce a change of 0.34 per cent in the value of the product pv.

Questions and Problems —1. How many significant figures should be retained in $\frac{1}{v}$ if four figures are properly retained in v?

2. What is the percentage deviation in pv due to the maximum observed change in temperature of the surroundings during the experiment?

BOYLE'S LAW.—II.

Object.—In this experiment the relation between the pressure and volume of a gas maintained at constant temperature is to be investigated at pressures greater than one atmosphere. A nearly perfect gas, dry air, and some imperfect gas, as ammonia or carbon dioxide, are those chosen for investigation. Practice is afforded in use of an open manometer for measuring pressures.

Apparatus.—The gases to be investigated are permanently sealed in the glass tubes *A* and *B* which are tightly screwed into a rectangular block of iron, soft stoppers being used for packing. An open manometer, *C*, is likewise fastened into a third opening in the block, all three tubes being freely connected with each other, as shown in the figure, and with a steel stop-cock, *S*, which connects the iron base with a reservoir of mercury, *R*. This is connected to a force pump by means of heavy pressure tubing, *T*.

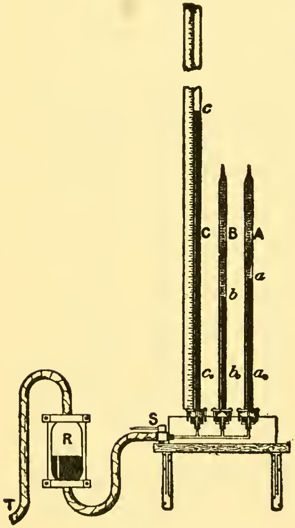

Fig. 25.

The glass tubes *A* and *B* are graduated from the top downward in *linear* centimeters and millimeters. The height of the mercury in the manometer is read off on a boxwood scale also graduated in millimeters. To facilitate reading the mercury column, the scale is grooved to fit the manometer tube. Accompanying the apparatus is a calibration curve for each of the tubes *A*, *B*, by means of which the volume of the gas in each tube can be obtained at once for any height of the mercury column.

Procedure.—Open the stop-cock S and *slowly* pump the mercury from the reservoir R into the system of tubes A, B, C, until it nearly reaches the top of the manometer C. Close the stop-cock S. This operation should be performed slowly, to prevent a sudden compression of the gases in A and B which will raise their temperature. The success of the experiment depends largely on maintaining the temperature of the gases as nearly constant as possible throughout. For this reason great care must be observed not to warm the portions of the tubes A and B containing the gas with the hands, breath, or otherwise during the experiment. Some temperature changes will undoubtedly occur in the gases owing to the change in volume to which they are subjected. It is therefore necessary to wait after each setting, particularly after the first compression, until the gas has assumed the temperature of the surrounding air, as indicated by a constant reading of the mercury columns in A and B. Usually it will be found that the gas has been heated during the first compression, and the pressure will be too great. As the gas then cools to the room temperature, the pressure will fall slightly as indicated by the rise of the mercury in the closed tubes. Final readings should only be taken after the mercury columns have assumed a constant position. When this is the case, read the height of the mercury columns in A, B, and C. Then open S and allow the mercury to flow back into the reservoir until the pressure has fallen in the open manometer about 30 cm. Close S and read A, B, and C as before. Repeat this procedure for not less than six different pressures, extending over as great a range as the apparatus will permit. Record also the reading of the barometer at the beginning and end of the experiment and of the thermometer suspended by the apparatus. Before leaving the experiment record also the "zero points" (see below) of the three tubes and obtain from the calibration curve the corrections necessary for reducing the observed readings to volumes.

Computation.—*To compute the pressure p.* The pressure which is exerted on the gases in the tubes A and B is evidently equal to the difference in height of the mercury columns in them and in C respectively, plus the pressure of the atmosphere exerted on the open end of C. The scales of A, B, and C, are all graduated from the top downwards, and if they were so adjusted that corresponding divisions were exactly in the same horizontal line, the difference in height of the columns would be given directly by the difference of the respective readings of the open and closed tubes. This condition is impractical and, indeed, unnecessary to fulfill as the result is readily obtained if the corresponding readings on A, B, and C at any point of the tubes are known. This may be obtained, once for all, by means of an accurately adjusted spirit level or a cathetometer, or better (and the way actually employed), by reading the height of the mercury columns in all three tubes when freely open to the air. Under these circumstances the mercury stands, of course, at exactly the same height in each tube, all being of the same internal diameter to eliminate effects of capillarity. Let the zero readings be a_0, b_0, c_0 for the tubes A, B, C respectively. Then the difference in height of the mercury columns in A and C, for example, corresponding to readings a, c respectively, will be

$$(c_0 - c) - (a_0 - a).$$

The actual pressure on the gas in A will be therefore,

$$p = H + (c_0 - c) - (a_0 - a),$$

where H is the height of the barometer expressed in the same units as a and c. As

$$H + c_0 - a_0 = k, \ a \text{ constant},$$

the expression $$p = k + a - c$$

is more convenient to use when several pressures are to be computed.

To find the volume v.—If the tube containing the gas were of uniform cross-section, and the zero graduation properly placed, the graduations would give the volumes directly in units depending on the volume corresponding to unit length of the tube. Neither of these conditions is exactly fulfilled so that a calibration of the tube is necessary. This is made once for all before setting up the apparatus. The cross-section of the particular tubes used is so chosen that the volume corresponding to one linear centimeter is approximately equal to one cubic centimeter. In the calibration curves accompanying the apparatus, the corrections to be added to the observed readings, expressed in centimeters, are plotted as ordinates, and the corresponding readings as abscissæ.

Compute and tabulate the value of p and of v for each set of observations, and also their product pv. For gases which strictly follow Boyle's Law, this product should be constant, and a plot made with values of pv as ordinates, and p as abscissæ should be a straight line parallel to OX. Inspection of the computed values of pv, will show that the law is approximately followed in the case of air, but quite widely deviated from in the case of ammonia. To bring out the deviations more clearly, plot values of pv as ordinates and p as abscissæ. The curves best representing the data will be found to be straight lines having negative tangents, but as the quantities of air and ammonia in the two tubes are not the same, a comparison of the tangents does not give a true measure of the relative deviations of the gases from a perfect gas. Find from the plot the equation of the line representing the data on ammonia gas.

Problems.—1. If the deviation in p is δ_p 0.5 mm., and the deviation in v is $\delta_v = 0.02$ cc., what is the resultant deviation in the product pv for the first observation of p and of v taken on air?

2. What is the resultant *percentage* deviation in this case?

3. How much will a rise of temperature of 2° C. affect the product pv at 20° C.? Compare Precision Discussion, p.89.

4. If the air is suddenly compressed adiabatically to one-third its volume, its initial temperature being 20° C., how much will its temperature be raised?

Note: For adiabatic compression or expansion,

$$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{\kappa-1},$$

where $\kappa = 1.41$ for permanent gases, and $T =$ absolute temperature.

LAW OF THE PENDULUM.

Object.—The object of this simple experiment is to determine by the graphical method the law connecting the time of vibration and the length of a simple pendulum, and to determine the value of g. The experiment gives practice in the use of a stop watch in timing vibrations, and the computation affords an excellent precision discussion.

Apparatus.—The apparatus consists of a spherical ball suspended by a very fine steel wire, the length of which can be varied by means of a windlass to which its upper end is attached. When the pendulum has been adjusted to the length desired, the wire is tightly held by a rigidly supported brass clamp. The length of the pendulum is then the distance from the lower edge of the support to the centre of oscillation of the ball. A stop watch registering 0.2 seconds is used for determining the time of vibration, and scales graduated in millimeters and calipers are provided for measuring the length of the pendulum and the diameter of the ball respectively.

Procedure.—Determine the time of vibration of the pendulum for five or six different lengths, varying from about one-half a meter to a meter and a half. This may be done with sufficient precision by counting the number of *single vibrations* for a period of not less than three minutes, using a stop watch reading to 0.2 seconds. Note that in counting the vibrations, the swing at the instant the watch is started is to be counted as *zero*, not as *one*. The swings should be timed when the pendulum is swinging with its maximum velocity, i.e., as it swings through the lowest part of its arc. This time can be estimated with much greater precision than when the pendulum is at the end of the arc, i.e., moving with zero velocity. Care should also be taken that the pendulum is set swinging in a plane, and not in an ellipse, and through an arc of not more than 10° or 15°. The length of the pendulum is to be measured with a millimeter scale, from the lower edge of the jaws clamping the wire, to the top of the ball. The diameter of the ball is to be measured with calipers. The measurements should in general be made as near as possible to tenths of a millimeter, except in the case of lengths of a meter or over, where the nearest half millimeter will suffice, for reasons shown under the precision discussion.

Computation.—Compute and tabulate the values of the time of vibration t, corresponding to each length l, of the pendulum. The length of the pendulum is to be computed by the expression,

$$l = h + r + \frac{2}{5}\frac{r^2}{h + r},$$

where h is the length of the wire from the point of support to the top of the ball, and r is the radius of the ball. The last term of the formula is the distance of the centre of oscillation below the centre of the ball. The computation is to be carried at each stage of the work to the nearest tenth of a millimeter.

First.—To deduce the law connecting t and l.

Suppose it is desired to deduce the functional relationship existing between the values of t and l, i.e., the form of the function $t = f(l)$. This can be done most conveniently by means of a graphical solution. If a "direct plot,"[1] be made with values of t as ordinates and l as abscissæ, the data will be found to lie along a curved line suggesting an exponential relationship of the general form $t = kl^n$, when k and n are constants. If such an equation holds, the values of the constants k and n can always be determined by the "logarithmic method."[2] Construct therefore a logarithmic plot, by plotting logarithms of t and l as ordinates and abscissæ respectively[3] on rectangular coördinate paper, and determine the equation of the resulting best representative line. From it deduce the desired constants in the equation $t = kl^n$.

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[1] See Graphical Methods, p. 41.

[2] Ibid., p. 52.

[3] Express l in meters.

Second.—To find the value of g from the plotted data.

The value of g may be computed at once from the value of k, as found above, if we assume the formula $t = \pi\sqrt{\frac{l}{g}}$ for the pendulum to be known. For this may be written $t = \frac{\pi}{\sqrt{g}} l^{\frac{1}{2}}$, which by comparison with the general equation, $t = kl^n$, shows that $n = \frac{1}{2}$ and $k = \frac{\pi}{\sqrt{g}}$, or $g = \frac{\pi^2}{k^2}$. Compute g in this manner.

Precision Discussion.—How precise should t and l be measured in order that the directly computed value of g shall be reliable to 0.2 per cent?

Solving by *equal effects* we have at once for the equation

$$g = \frac{\pi^2 l}{t^2},$$

$$\frac{\Delta_l}{g} = \frac{\Delta_t}{g} = \frac{1}{\sqrt{2}} \cdot \frac{\Delta}{g} = \frac{1}{\sqrt{2}} \times 0.002 = 0.0014.$$

But $\quad \frac{\Delta_l}{g} = \frac{\delta_l}{l}$ and $\frac{\Delta_t}{g} = 2\frac{\delta_t}{t}$.

Hence $\frac{\delta_l}{l} = 0.0014$ or l must be measured to 0.14 per cent,

and $\quad \frac{\delta_t}{t} = 0.0007$ or t " " " " 0.07 per cent

If the pendulum is a seconds pendulum, i.e., one meter in length, the time of one vibration must be known to 0.07 per cent of one second or 0.0007 second; and the computed length l must be known to 0.14 per cent of 1000 millimeters or 1.4 mm. For a pendulum only one-quarter of a meter in length and beating therefore half-seconds, the required precision will be 0.14 per cent of 250 mm. or 0.35 mm. in the length, and 0.07 per cent of 0.5 second or 0.00035 second in the time. This precision can be attained without great difficulty with the apparatus provided.

Questions and Problems.—1. About what do you estimate the precision of the constants k and n as determined from the plot to be?

2. Do you think your data would warrant further discussion by means of a "residual plot"? (See Notes on Graphical Methods, p. 60). Why?

3. Is the correction term $\frac{2}{5}\frac{r^2}{h+r}$ negligible in the computation of any values of l? Why?

THE METHOD OF COINCIDENCES.

GENERAL DISCUSSION.

The two following experiments involve a more accurate determination of the time of vibration of a pendulum than in the preceding experiment, and the method used, known as the "Method of Coincidences," is of great importance and wide application. The following discussion should therefore be carefully studied before performing either of the experiments in question.

We have given two pendulums A and P, of approximately the same time of swing, that of A being unknown, while that of P is known. It is desired to find the rate of A in terms of P.

Suppose first, that both pendulums A and P have exactly the same time of vibration. If both are started swinging in the same phase, they will continue in phase, both passing through the lowest point of their respective arcs simultaneously. If at these points contact devices, for example mercury cups, are placed by which an electric circuit is closed when both pendulums make contact simultaneously, the time of passage will be indicated by the stroke of a bell, sounder, or other recording device placed in the circuit. If the pendulums are started swinging in exactly opposite phase, they will also continuously pass through the lowest part of their arcs at the same instant. If they start in anything except the same or opposite phase, they will never come into phase or coincidence, and the circuit will never be closed. Suppose however, that the time of vibration of A is slightly greater than that of P and they are started swinging in the same phase. A will immediately begin to lag behind P, and hence both pendulums will, after the first swing, no longer pass through their respective mercury cups at the same instant; the circuit will therefore not be closed, and no signal will be given. The amount by which A lags behind P will increase continually at each swing until it is just one single vibration behind P, i.e., until it comes into coincidence but in opposite phase with P, when again both pendulums will simultaneously close the circuit at the mercury cups, and the event will be indicated by the recording device. If the number of vibrations of the standard pendulum P, elapsing between these two successive "coincidences" be n, then while P has made n vibrations, A will have made $n-1$ vibrations; consequently if P is a seconds pendulum, the time of vibration of A will be $\frac{n}{n-1}$ seconds. Conversely if the pendulum A has a shorter time of vibration than P, A will make $n+1$ vibrations while P makes n vibrations, and the rate of A will be $\frac{n}{n+1}$ seconds.

If the rate of the standard pendulum P is not exactly known, it must be determined by a comparison with a standard clock, chronometer, or watch. If its rate, i.e., its time of vibration in true seconds is found to be r, then the rate of A, as determined above, must be multiplied by r.

Precision Discussion.—From the preceding discussion, it follows that the time of vibration determined by the method of coincidences is given by the expression $t=\frac{n}{n\pm 1}$, where n is the time elapsing between successive coincidences. Let us find what deviation Δ is introduced in t by a deviation δ_n in n. Differentiating the expression for t with respect to n, we have

$$\Delta=\frac{n\pm 1-n}{(n\pm 1)^2}\delta_n,$$

$$=\pm\frac{\delta_n}{(n\pm 1)^2}.$$

Hence $$\frac{\Delta}{t}=\pm\frac{1}{n\pm 1}\frac{\delta_n}{n},$$

that is, the fractional or percentage deviation in t is only the $\frac{1}{n \pm 1}$ part of that in n. Thus if the deviation measure of the time of coincidence is one per cent, the resulting value of t will be uncertain by only $\frac{1}{n \pm 1}$ of one per cent. It is evident, therefore, that the method gains in precision the greater the time of coincidence, i.e., the smaller the difference in the rates of the two beating pendulums.

To illustrate: Suppose $n = 100$ seconds and $\delta_n = 1$ second; then $\frac{\delta_n}{n} = \frac{1}{100}$, i.e., n is uncertain by one per cent, while t, on the other hand, is uncertain by only $\frac{1}{100 \pm 1}$ of one per cent.

If $n = 50$ seconds and $\delta_n = 1$ second, the percentage deviation in n will be two per cent, but the percentage deviation in the resulting value of t will be only $\frac{1}{50 \pm 1}$ of two per cent or 0.04 per cent. The method is thus seen to be one of extreme precision.

THE PHYSICAL PENDULUM.

Object.—This experiment is designed to measure the value of g to about 0.1 per cent. It illustrates the "method of coincidences" in measuring time, and also a method of measuring lengths greater than the length of the measuring scale with the aid of two microscopes. The experiment consists of two parts: *first,* the determination of the time of vibration of the pendulum; and *second,* the measurement of its length.

I.—DETERMINATION OF TIME OF VIBRATION.

Apparatus.—The pendulum consists of a brass sphere A fixed near the lower end of a brass rod, which is supported on a steel knife edge, N. Above the knife edge the rod carries a smaller brass sphere, B. The whole constitutes of course a metronome pendulum; but the position of the sphere

A is so adjusted once for all, that the time of vibration of the pendulum as a whole, is equal to that of an equivalent simple pendulum, consisting of the ball *A* suspended by a weightless thread at the same distance below the knife edge. This is accomplished as follows: The ball *A* being removed from the rod, the time of vibration of the latter is accurately determined, the smaller ball *B* being so adjusted above the knife edge that the time of vibration is approximately one second. The ball *A* is then replaced on the rod and adjusted by trial until a position is found such that the time of vibration is the same as with *A* removed. *A* is fixed in this position and the whole pendulum may then be regarded as a simple pendulum, the length of which is the distance of the centre of oscillation of the ball *A* below the knife edge. This adjustment has been carefully made and is not to be disturbed.

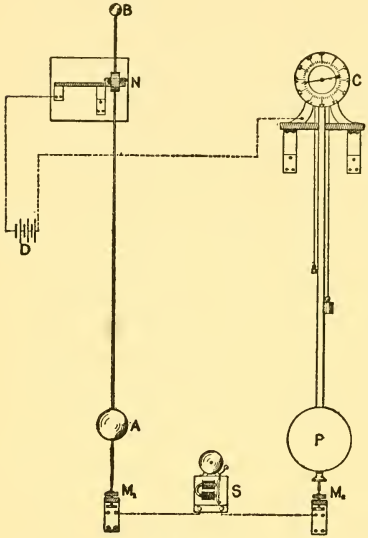

Fig. 26.

The rod of the pendulum terminates in a platinum point which swings through a mercury globule M_1. By the side of the pendulum is mounted a clock *C* beating seconds, the pendulum of which is also provided with a platinum point which swings through an adjustable mercury cup M_2. The two pendulums are connected in series with a battery *D* and an electric bell *S* (or sounder), so that when contact is made at M_1 and M_2 simultaneously, the circuit is closed and the bell (or sounder) responds.

Procedure.—With P at rest and making contact through M_2, adjust the mercury contact M_1 until the bell strikes regularly at every swing of A. The adjustment should be so made that the platinum point just cuts through the top of the mercury drop. Next, with A at rest, adjust M_2 so that when P is set swinging, contact is made regularly at each vibration. Finally set both pendulums swinging through an arc of about 10°, taking care that they swing in a plane and not an ellipse. The pendulum A is best set swinging in a plane by drawing it to one side with a thread and burning the thread. The clock pendulum can be set swinging with sufficient accuracy with the hand. If the adjustments have been well made the bell or sounder should strike not more than two consecutive times when the pendulums come into coincidence. The reason why more than one coincidence is often observed is that one pendulum gains or loses on the other so little in one or even two vibrations, that both pendulums are able to touch some portion of the mercury contacts, which are of sensible width, at the same instant for several consecutive beats. When the pendulums are swinging properly, note *by the clock* the time of coincidence, recording the time of first and last coincidence in case the bell strikes more than once. The mean is to be taken as the true time of coincidence. The time of about nine consecutive coincidences should be recorded. Then start the pendulums swinging again and take a second similar series.

To complete the time observations it remains only to determine the rate of the clock which should not be assumed to beat seconds without further test. The rate may be obtained with sufficient accuracy by making a fifteen minute comparison with a watch. Wind up the clock and with watch in hand (which must be provided with a seconds hand), note the exact time in seconds of the watch corresponding to the time in seconds of the clock. This is best done by keeping the clock time in mind by listening to the tick, while observing at the same time the seconds hand of the watch. Record the second of the clock and watch when both tick together. No further attention need then be paid to the clock for about fifteen minutes. At the end of this time note as before the time of the clock and keep time mentally by counting its beats while looking at the seconds hand of the watch. At the end of exactly fifteen minutes by the clock note the second and fraction of a second, if possible, of the watch. If, for example, it is found that fifteen minutes by the clock is equal to fifteen minutes plus one and one-half seconds by the watch, the rate of the clock is

$$r = \frac{901.5}{900} = 1.0017 \text{ seconds.}$$

II.—MEASUREMENT OF LENGTH OF PENDULUM.

Apparatus.—The apparatus required for measuring the length of the pendulum consists of a horizontal frame for holding the pendulum, a steel scale graduated in millimeters which can be adjusted in the plane of the knife edge of the pendulum, and two reading microscopes.

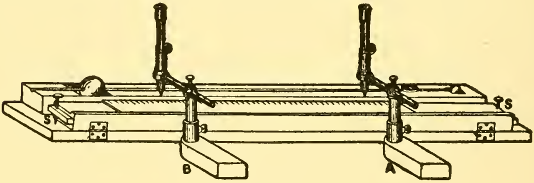

Fig. 27.

Procedure.—Remove the pendulum from its support and place it in the frame, taking care in carrying it *not to bend the pendulum rod.* Carefully focus the microscopes *A* and *B*, so that the intersection of the cross hairs comes exactly on the knife edge, and on the point where the rod enters the top of the ball respectively. The length it is desired to measure is then the distance between the cross wires of the two microscopes; the greatest care must therefore be taken not to displace them after once focused. Next move the frame to one side so that the scale is seen under the microscopes instead of the pendulum. It will, in general, be found to be too short to be seen under both at the same time. If it is not in good focus, raise or lower it by means of the adjusting screws S, at ends of the frame until the divisions can be clearly seen without parallax in either microscope.

When the scale is focused, move it along until some division a, exactly coincides with the cross hairs in microscope A. Then, without moving the scale, move microscope A towards B, and set it so that the intersection of the cross hairs comes on some division b. It has been moved towards B through a distance $a - b$. Now leaving A fixed in position, slide the scale along towards B until a third division, c, coincides with the cross hairs in A, and finally read the position of the cross hairs in B on the scale to tenths of a millimeter, and call this reading d. The length of the pendulum from the knife edge to the top of the ball is then

$$h = (a - b) + (c - d).$$

Repeat the measurement of h and take the mean. The radius r of the ball is to be found from the mean of several measurements of its diameter, taken with calipers. All measurements should be made to tenths of a millimeter.

Computation.—The distance of the knife edge to the centre of oscillation of the pendulum is to be computed from the above measurements by the formula,

$$l = h + r + \frac{2}{5}\frac{r^2}{h + r}.$$

Compute the mean time of coincidence and its percentage deviation. Calculate the time of vibration t, of the pendulum, its percentage deviation and its numerical deviation in seconds. (See Precision Discussion, p. 98.) With this value of t (corrected for rate of clock) and the value of l, compute g.

Questions and Problems.—1. How closely should t and l be measured if it is desired to obtain the value of g to 0.1 per cent?

2. What is the percentage deviation of your computed value of t? What is its deviation in seconds?

3. Assuming $\delta_l = 0.2$ mm. and $\delta_t =$ the value computed in Problem 2, find the resultant deviation in the value of g and also its percentage deviation.

METRONOME PENDULUM.

The discussion of the "Method of Coincidences," p. 97, should be read before performing this experiment.

Object.—This experiment affords practice in determining the time of vibration of a pendulum by the method of coincidences, the coincidences being observed by a reflected beam of light, in the use of a telescope and scale for measuring vertical distances, and in the use of the vernier caliper. The data serve to verify the formula for the time of vibration of a metronome pendulum.

Apparatus.—The pendulum consists of two brass spheres, A and B, Fig. 28, the smaller, A, being fixed near the upper end of a solid brass rod, and the larger, B, near the lower end. The pendulum swings on a steel knife edge C, adjusted at the centre of gravity of the rod. Just above the knife edge a small plane mirror M is attached to the rod. By the side of the pendulum a steel scale not shown in the figure is suspended for measuring its length. The apparatus for observing the time of vibration of the pendulum is shown in Fig. 29. T is a reading telescope provided with cross hairs at the focus of the eye-piece. It is adjustable to any height and inclination. The apparatus shown above the telescope is a device for sending a flash of light to the mirror on the pendulum once every second. The light from any brilliant source, e.g., an incandescent lamp, is reflected by a mirror N through the lens L by means of which it is focused on a horizontal slit S_1. Behind S_1 is placed a second horizontal slit S_2, attached to the end of a lever connected with the armature of an electromagnet. The slits are so constructed and adjusted that each time the armature is attracted to the magnet, the slit S_2 uncovers the slit S_1 and allows a flash of light to pass to the pendulum beyond. This is made to occur once every second by connecting the electromagnet in series with the pendulum of a standard clock beating seconds, so that the circuit is made once a second. The flash is observed by focusing the telescope on the slit S_1, as seen reflected in the mirror M of the pendulum. If the pendulum is at rest and the magnet set in operation, a flash will be seen in the telescope once every second. If the pendulum is swinging, the flash will occur only when the pendulum and the clock come into "coincidence."

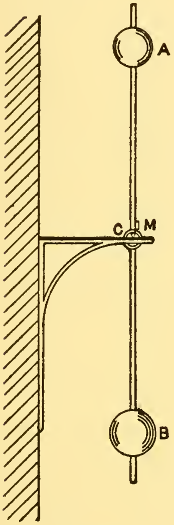

Fig. 28.

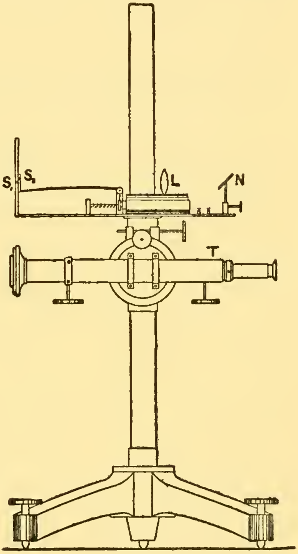

Fig. 29.

Procedure.—*Adjustment.*—Set up the telescope about one meter from the pendulum and clamp the slit a little above, and the telescope T a little below the height of the mirror M. Focus the eye-piece carefully on the cross hairs. Then with the pendulum at rest, turn the telescope towards the mirror M and focus on the slit S as seen in the mirror. This is most readily done by holding a lighted match or candle in front of S and adjusting the slit and the telescope until the former is clearly seen. It is then easy to complete the adjustment so that the image of the slit is seen on the horizontal cross hair of the eye-piece. When the telescope is thus focused, close the circuit through the electromagnet, and adjust the mirror N and the lens L until a bright flash of light is seen in the telescope at each beat of the clock. The apparatus is then in adjustment.

Measurement of time of coincidence.—Set the pendulum swinging through a very small arc, and look for the flash in the eye-piece of the telescope, when the clock and pendulum come into coincidence. Several flashes will be seen alternately above and below the central cross wire but always approaching it until the point of coincidence is reached. The time when the flash occurs on the cross hair is to be taken as the true time of coincidence. The time elapsing between successive coincidences is easily counted by the clicks of the armature of the electromagnet. Record the time of about nine consecutive coincidences, then start a new series. The rate of the clock may be assumed zero.

Measurement of the dimensions of the pendulum.—It is necessary to measure the distance of each ball above and below the knife edge C, the diameter of each ball, and the length and diameter of the rod. Suspend a steel scale beside the pendulum in the plane of the knife edge. Raise the slit apparatus out of the way to the top of the telescope stand. Level the telescope and focus it on the top of the rod. The telescope should be placed at such a distance from the pendulum that both rod and scale appear in the field of view at the same time. Adjust until the horizontal cross hair is coincident with the top of the rod and read its height on the scale to 0.1 millimeter by estimation. In the same way take readings of the height of the top and bottom of both balls and of the knife edge. Repeat the measurements if time permits.

Measure the diameter of the rod with the vernier calipers provided. The masses of the various parts of the pendulum are given with the apparatus.

Computation.—*First.*—Compute the true time of vibration of the metronome pendulum from the data on coincidences, as explained on p. 98.

Second.—Compute from the dimensions and mass of the pendulum the length of the equivalent simple pendulum having the same time of vibration. The formula for this reduction is

$$l = \frac{\text{sum of moments of inertia of moving parts about } C}{\text{sum of statical moments about } C}.$$

Let l_1 = distance of centre of gravity of A above C;
l_2 = " " " " " " B below C;
m_1 = mass of A;
m_2 = " " B;
m = " " rod;
l = half the length of the rod;
r = radius of the rod.

Then the value of l becomes

$$l = \frac{m_1 l_1^2 + m_2 l_2^2 + m(\frac{r^2}{4} + \frac{l^2}{3})}{m_1 l_1 + m_2 l_2}.$$

The statical moment of the rod which ordinarily enters in the denominator is here zero as the knife edge C passes through the centre of gravity of the rod. For a demonstration of above formula, see Lanza's Applied Mechanics.

With the value of l computed by this formula, compute the corresponding time of vibration of the equivalent simple pendulum assuming $g = 980.4$. Compare with the value experimentally found, and state the percentage deviation between them.

Question.—1. State all sources of error which you recognize in this experiment.

LAW OF FREELY FALLING BODIES.

TUNING-FORK CHRONOGRAPH.

Object.—The object of this experiment is to give practice in the measurement of small intervals of time (a second or less) by means of the tuning-fork chronograph. The use of this instrument is illustrated by the determination of *g*, the acceleration due to gravity, by a freely falling body, i.e., by measuring the time required for a body to fall through a given distance. The discussion of the results gives practice in the use of the logarithmic method of plotting, and incidentally the experiment illustrates the method of recording the occurrence of an event by an electric spark, and the method of estimating tenths of an harmonic vibration.

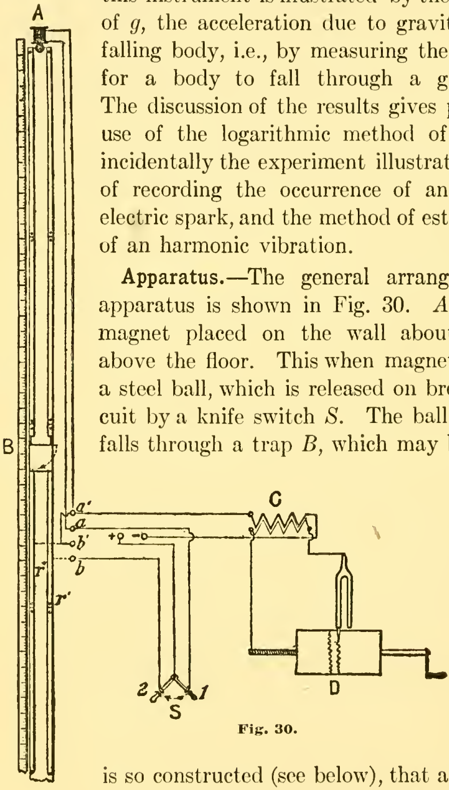

Fig. 30.

Apparatus.—The general arrangement of the apparatus is shown in Fig. 30. *A* is an electromagnet placed on the wall about sixteen feet above the floor. This when magnetized holds up a steel ball, which is released on breaking the circuit by a knife switch *S*. The ball in its descent falls through a trap *B*, which may be adjusted at any desired distance below *A*, the distance *AB* being read off on a fixed boxwood scale graduated in millimeters. The trap *B* is so constructed (see below), that a second circuit is broken the instant the ball passes through.

The apparatus for recording the interval of time elapsing between the breaking of the circuit at *A* and at *B*, consists first, of an induction coil *C*, the primary coil of which is connected in series first with *A*, and immediately afterward with *B*. When the circuit through *A* or *B* is broken, the current induced in the secondary of the coil causes a spark to pass across its terminals. These sparks are recorded in the manner described below on a revolving drum *D*. Since the sparks occur practically simultaneously with the releasing of the ball and the opening of the trap, a measurement of the time interval between them gives the time desired. This method of recording two events electrically has the great advantage that the simultaneously occurring secondary event, the sparking, can be made to occur at any place convenient for measuring or recording, while the primary event itself may take place at any remote distance.

The interval elapsing between two consecutive sparks may be easily and accurately recorded as follows: A cylindrical metallic drum is so mounted on a horizontal screw, that on turning the latter, the drum is given a combined rotary and lateral motion.

The drum is covered with a sheet of thin paper which is evenly coated with lampblack. One terminal of the secondary of the induction coil is connected through the bearings to the cylinder, the other terminal is connected to a metallic style resting on the paper. A spark, on passing between the style and the drum, punctures the paper and "splashes" the lampblack at the point of contact of the style, in the form of a white dot. If the drum be rotated under

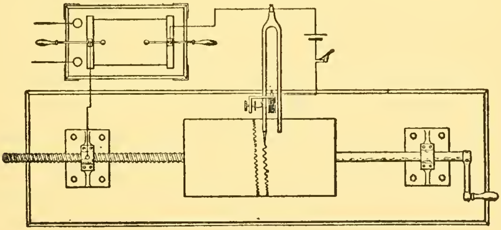

Fig. 31.

the style, a straight line will be traced on the lampblack, and sparks or events will be recorded as dots on a smooth line. If the drum were rotated uniformly at a known rate, the time between any two events could be determined. This condition would be very difficult to realize, but may be got around by causing a tuning-fork of known pitch to simultaneously record its vibrations on the drum. This may conveniently be done by fixing the style (a piece of very thin spring), to one prong of a tuning-fork, as in Fig. 31. The fork being now connected to one terminal of the coil and set into vigorous vibration, and the drum rotated, a sinusoidal curve will be traced on the drum, and the events marked by the sparks will now be recorded on a sine curve, each wave of which represents a known interval of time. The time between two events is found by counting the number of vibrations and fractions of a vibration of the fork between the corresponding dots. The result is independent of the velocity of rotation of the drum, a greater velocity serving merely to draw out the waves to a greater extent, but in no way altering their number. It is convenient to have the fork driven electrically by a small electromagnet placed between the prongs and run on an independent battery circuit, as shown in Fig. 31. The amplitude of vibration may thus be maintained constant and vigorous for any length of time.

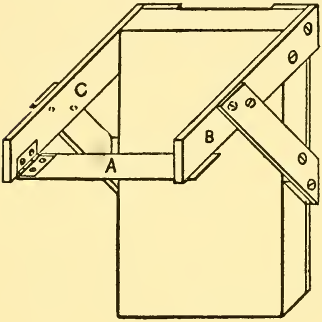

Fig. 32.

The trap *B* (Fig. 30), consists of a flat strip of iron *A* (Fig. 32), hinged at one end and held up horizontally in the path of the falling ball by a strong permanent magnet *B*. The current passes across from *B* through *A* to *C*, so that the instant the ball falls upon *A* the circuit is broken. The electrical connections will be easily seen from Fig. 30.

When the circuit through the electromagnet is closed, the current passes from the positive terminal of the storage battery to 1, a, A, a', C, back to the negative terminal. On throwing the jack-knife switch from 1 to 2, the circuit is broken, but instantly afterward closed again through the trap B, the current then passing from the positive terminal to 2, b, up the metallic rail r' to B, across the trap to the other rail r'' down to b', a', C, and so back to the negative terminal.

Procedure.—*Preparation of lampblack cylinder.*—Fasten a sheet of the special chronograph paper tightly over the drum. See that the edges of the paper overlap in such a manner that on rotating the drum the style passes from the upper to the under, otherwise it is likely to catch and spoil the record. Hold a piece of burning camphor in the spoon provided *close* under the cylinder, and revolve the cylinder back and forth over the smoky flame until uniformly covered with a thin coating of soot. The *deposit should not be too thick* to secure the best results. An even brown deposit is much better than a heavy black one, as the spark records are much better defined on the former, and it is, moreover, much easier to "fix," as explained below.

Preliminary trial.—First: Close the battery circuit and switch S on 1; place the ball at A; see that B is closed and placed at some convenient distance below A. Rest the point of the style lightly against the drum, and while rotating the latter slowly, quickly pull open the switch S. Inspection will show that this is so constructed that the one act of pulling it open, instantly after closes the circuit through B. Two sparks should leave their records upon the drum. If they do not, look over the connections carefully, and repeat until this result is obtained.

Second.—Close the auxiliary battery circuit through the tuning-fork, and adjust the contact so that on striking the fork it continues to vibrate vigorously. Adjust it so that the style rests *lightly* on and at right angles to the drum, and rotate the latter so as to determine the speed at which the vibrations are drawn out into a nearly normal sinusoid.

Third.—Repeat the first adjustment with fork vibrating. If the two sparks are clearly recorded on the sine wave, the apparatus is ready for use.

Experiment proper.—Determine the time it takes the ball to fall through distances about 1, 1½, 2, 3, and 4 meters, making three determinations of the time for each distance. It is best to locate with a pencil the position of the two sparks immediately after each experiment, marking the first 1 and the second 2. To facilitate the subsequent estimation of tenths of a vibration, a straight line should be traced through the centre of the sinusoid with the tuning-fork at rest. When the cylinder has become covered with records, they may be "fixed" with a fixatif sprayed from an atomizer, and the paper removed for future reduction, or they may be "reduced" at once.

It is interesting to use several balls of different diameters in this experiment to verify Galileo's experiment on falling bodies.

Measure the diameter of the ball with calipers.

Record data given with the apparatus on the position of the electromagnet A and the true rate of the fork. This has been accurately rated while being driven electrically under the conditions of the experiment against a standard fork.

Computation.—The distance s, through which the ball falls, is the difference in height as read off on the vertical scale, of the lower end of the core of the electromagnet and the upper surface of the trap, minus the diameter of the ball. The height of the former has been accurately determined once for all, and is given on the apparatus.

Reduction of time.—The interval of time elapsing between the two events may be determined to one vibration of the fork by counting the number of complete vibrations between the two recorded points. In general, the two events do not occur in the same phase of the vibration, so that to obtain the accuracy of which the method is capable, it is necessary to estimate fractions of a vibration. As the motion of the fork is simple harmonic, equal distances on the sinusoid do not correspond to equal differences of time. The estimation or measurement of tenths of a vibration must therefore be made as follows:—

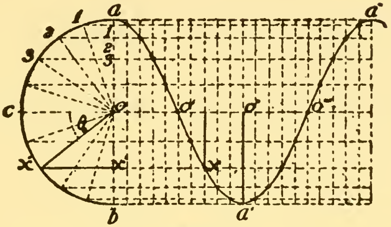

Fig. 33.

Suppose *ab* represents the amplitude and actual path of vibration of the point of the style, *o* being its position of rest. It vibrates with simple harmonic motion in this line, and traces on the drum revolving at right angles to it the sinusoidal curve $a\ a'\ a''$.

If the fork be considered to start vibrating from its extreme position *a*, its position at any time *t* will be found by projecting on *ab*, the position of an imaginary point revolving in the circle *acb* with a uniform angular velocity $\frac{\pi}{T}$, where *T* is the time of one single vibration of the fork. At times $\frac{1}{10}\frac{\pi}{T}$, $\frac{2}{10}\frac{\pi}{T}$, $\frac{3}{10}\frac{\pi}{T}$, etc., the imaginary point and the style will be at points 1, 2, 3, etc., on the circle *acb* and line *ab* respectively, while the corresponding points on the curve traced by the style will be the dots indicated. Thus it is evident that equal intervals of time mark off different distances on the sine curve, and that the estimation of tenths of a vibration in the ordinary manner would lead to entirely false results. The estimation of tenths of a vibration in the reduction of experimental results is to be made by comparing the records obtained with the position of the points indicated in Fig. 33, or with the larger plot provided with the apparatus. When a higher degree of accuracy than can be obtained by estimation is desired, the procedure is as follows: Suppose it is desired to determine the time corresponding to the position of a point x on the sine curve. If we reckon time as is usual with a vibrating body from the moment of zero elongation, i.e., when the body is moving through its position of equilibrium, in this case through o, o', o''', etc., the time corresponding to the distance $o'x$ is equal to the time it takes the style to vibrate from o to x', or for the imaginary point to pass from c to x''. But

$$ox' = ox'' \sin\theta = ob \sin\frac{\pi}{T}t,$$

$$\therefore \quad \frac{\pi}{T}t = \sin^{-1}\frac{ox'}{ob}; \; t = \frac{T}{\pi}\sin^{-1}\frac{ox'}{ob}.$$

Hence the ratio of the ordinate of the given point x to the maximum ordinate of the sine curve, is the sine of an angle, which multiplied by the constant $\frac{T}{\pi}$ gives the desired time. The ordinates of x and a' are best measured on a dividing engine.

First.—Compute and tabulate values of g for each set of observations. Compute the mean value of g, also its *A.D.* and percentage deviation.

Compute the *a.d.* of all time measurements. This will give a measure of the degree of precision of the above chronograpic method.

Second.—The formula $s = \frac{1}{2}gt^2$ may also be tested by discussing the data graphically by means of a logarithmic plot. Unless the direct plot be followed by a *residual* plot however, some of the precision of the data will be sacrificed, so that the value of g computed from the plot will not be as reliable as that computed by the formula.

Questions and Problems.—1. What must be the percentage precision of s and of t in order that the computed value of g may be reliable to 0.1 per cent? Is this degree of precision attainable with the apparatus used? If not, why?

MODULUS OF ELASTICITY.

Object.—The object of this experiment is, *first*, to determine the law connecting the deflection of a beam supported at its ends and loaded at its centre, with its length; and *second*, assuming the formula for the deflection known, to calculate from the measurements obtained, the modulus of elasticity of the beam.

Apparatus.—The apparatus (Fig. 34) consists of two supports *AA*, carrying knife edges on which the beam is supported. The distance between them may be varied and accurately measured on a fixed steel scale. The bar to be investigated rests on its narrower side on the supports. The deflection at the centre of the beam when loaded by a weight *W*, is measured by a fine micrometer screw *S* connected in series with a battery *B* and a galvanoscope *G*. The pitch of the micrometer screw is one millimeter. Inspection will show that when the end of the micrometer screw makes contact with the bar, the circuit will be closed and a deflection of the indicating magnet thereby produced. This serves as a very sensitive method of noting the instant of "contact." The breadth and depth of the bar are to be measured by means of Brown and Sharpe micrometer calipers.

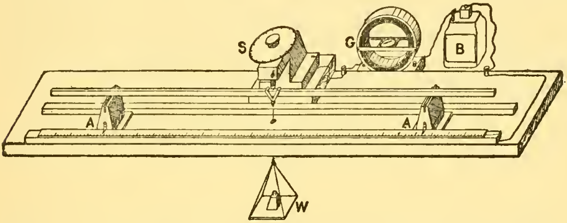

Fig. 34.

Procedure.—*First.*—Measure the deflection of the bar produced by a given weight *w*, with lengths, *l*, between the supports, equal to 100, 90, 80, 70, and 60 cm. respectively.

In each case make at least four independent settings of the micrometer, first with no load, and second, with the applied load. The deflection a, is to be taken as the difference between the means of these observations.

At the conclusion of the deflection measurements remove the load from the bar.

Second.—Measure the mean depth d and the mean breadth b of the beam with the calipers provided, taking nine independent measurements of each at different parts of the beam.

Computation.—*First.*—Compute the percentage deviation of the mean results of b and d, and of the maximum value of a.

Second.—Compute the value of the modulus of elasticity E of the beam for each set of measurements, in kilograms per square millimeter. Reduce the average value thus obtained to pounds per square inch. The formula for the deflection of a rectangular beam loaded at its centre is

$$a = \frac{wl^3}{4\,Ebd^3}.$$

Third.—Assuming the error in the weight w to be negligible, compute the resultant percentage deviation in the value of E obtained from the measurements with the length of beam equal 100 cm. Assume the deviation in l to be 0.2 per cent.

Fourth.—Determine graphically the law connecting the deflection a and the length l of the beam.

If a plot be constructed with values of a as ordinates and l as abscissæ, it will be found to give a curve suggesting an exponential relation between a and l of the form $a = kl^n$, where k and n are constants. To test this, and at the same time to determine k and n graphically, construct a "logarithmic plot"[1] with values of log a and log l as ordinates and abscissæ, respectively. Determine k and n from the plot.

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[1] See Graphical Methods, pp. 52 to 58.

The same results may also be obtained by making a direct plot of a and l on logarithmic plotting paper.

[Note.—The values of a and of l should be expressed in such units that the plotted data of the logarithmic plot cuts the axis of ordinates without a long extrapolation, as will be found to be the case if both a and l are expressed in millimeters. If however, l is expressed in meters, the locus of the data will be shifted parallel to itself towards the Y-axis, the value of $\log a$ corresponding to $\log l = \log 1$ (meter) $= 0$ falling on the axis. As the inclination of the line remains unchanged the value of $n = \tan \theta$ remains also unchanged. The intercept will be however $n \log 1000 = 3n$ units greater than it would be if both a and l were expressed in millimeters. For let

$$\log a = n \log l + \log k \tag{1}$$

be the equation of the line with a and l both expressed in millimeters and

$$\log a = n \log l' + \log k' \tag{2}$$

be the equation when a is expressed in millimeters and l in meters. Since

$$l = 1000\, l',$$

(1) may be written

$$\begin{aligned} \log a &= n \log (1000\, l') + \log k, \\ &= n \log l' + 3\, n + \log k, \end{aligned}$$

which compared with (2) gives

$$\log k' = 3\, n + \log k,$$

i.e., the intercept on the Y-axis is $3n$ units above the intercept in the first case.]

Fifth.—From the value of k obtained from the plot compute the value of the modulus of elasticity E, and compare it with the values previously obtained.

Since $$a = \frac{w}{4Ebd^3} l^3, \text{ and } a = kl^n,$$
it is seen that
$$k = \frac{w}{4Ebd^3},$$
from which E may be readily computed.

Precision Discussion.—The above experiment affords an excellent example of an indirect measurement involving a number of components, five in this case. Suppose the precision desired in E to be Δ units or the fractional precision to be $\frac{\Delta}{E}$; how precise must each of the components w, a, l, b, and d be measured?

Solving by equal effects we must have
$$\frac{\Delta_w}{E} = \frac{\Delta_a}{E} = \frac{\Delta_l}{E} = \frac{\Delta_b}{E} = \frac{\Delta_d}{E} = \frac{1}{\sqrt{5}} \frac{\Delta}{E},$$
from which it follows at once, that the fractional precision of the several components must be as follows:—
$$\frac{\delta_w}{w} = \frac{\Delta_w}{E} = \frac{1}{\sqrt{5}} \frac{\Delta}{E}; \quad \frac{\delta_a}{a} = \frac{\Delta_a}{E} = \frac{1}{\sqrt{5}} \frac{\Delta}{E}; \quad \frac{\delta_b}{b} = \frac{\Delta_b}{E} = \frac{1}{\sqrt{5}} \frac{\Delta}{E};$$
$$3\frac{\delta_l}{l} = \frac{\Delta_l}{E} \therefore \frac{\delta_l}{l} = \frac{1}{3} \frac{1}{\sqrt{5}} \frac{\Delta}{E}; \quad 3\frac{\delta_d}{d} = \frac{\Delta_d}{E} \therefore \frac{\delta_d}{d} = \frac{1}{3} \frac{1}{\sqrt{5}} \frac{\Delta}{E}.$$

Of the five quantities measured, w can usually be determined with ease to a precision far greater than any of the others, so that the deviation in the final result arising from it may be considered negligible. In this case the number of variables is reduced to four and the allowable deviation in each component may be a little greater for a prescribed precision in the final result. Note that a problem of this kind should always be solved using fractional or percentage deviations and not actual deviations as the labor thus saved in the computation is very considerable.

Questions and Problems.—1. Do your computed values of E show any regular deviations with decreasing values of l, and if so, how do you explain it?

Optional Problems.

2. What is the allowable percentage deviation in l, a, b, and d if the value of E is desired to 0.5 per cent? Assume the deviation in w negligible.

3 For the given specimen, 1000 mm. long, what would be the allowable deviations in millimeters in l, a, b, and d corresponding to the condition imposed in problem 2?

4. Do you think this precision was attained in your results, and why?

APPENDIX.

REFERENCE TABLES.

Physikalisch-Chemische Tabellen, Landolt and Börnstein. Very extensive and reliable data of physical and chemical constants.

Mathematical and Physical Tables, Wrapson and Gee.

The Constants of Nature, Clarke.

Smithsonian Meteorological Tables.

TABLE I.

FORMULÆ AND MATHEMATICAL CONSTANTS.

Circumference of circle $= 2\pi r$
Area of circle $= \pi r^2$
Simpson's Rules for Areas, see page 10
Surface of sphere $= 4\pi r^2$
Volume of sphere $= \frac{4}{3}\pi r^3$
Ratio of circumference to diameter of circle, $\pi = 3.141593$
$\pi^2 = 9.869604$
$1/\pi = 0.3183099$
$\log \pi = 0.4971499$
Base of Napierian logarithms $e = 2.7182818$
$\log_{10} e = 0.4342945$
$1/\log_{10} e = 2.302585$
1 Radian or arc equal to radius $= 57° \, 17' \, 45''$
$= 206265''$

TABLE II.

NUMERICAL RELATION BETWEEN VARIOUS UNITS.

ENGLISH AND METRIC MEASURES.

Note.—Values taken from "Tables of Weights and Measures," U. S. Coast and Geodetic Survey, 1890.

LENGTH.

1 meter = 39.3700 inches (legalized ratio for the U. S.)
1 meter = 1.093611 yard
1 meter = 3.280833 feet
1 kilometer = 0.621370 mile
1 inch = 25.40005 millimeters
1 foot = 0.304801 meter
1 yard = 0.914402 meter
1 mile = 1.609347 kilometer

TABLE II.—Continued.

MASS.

1 kilogram = 2.204622 pound av.
1 gram = 15.43235639 grains
1 pound = 0.4535924277 kilograms
1 ounce av. = 28.34853 grams
1 ounce troy = 31.10348 grams

VOLUME.

1 liter = 1.05668 quarts
1 liter = 0.26417 U. S. gallon
1 liter = 33.814 U. S. fluid ounces
1 quart, U. S. = 0.94636 liter
1 gallon, U. S. = 3.78544 liters
1 fluid ounce = 0.029573 liter

TIME.

Mean length of civil year = 365 days, 5 hours, 48.8 minutes
Sidereal day = mean day minus 3 minutes 55.9 seconds
= 0.99727 mean day

MECHANICAL EQUIVALENT OF HEAT.

1 kilogram-calorie (1 kilogram water raised 1° C. at 15° C.) = 427.3 kilogrammeters (at sea-level, latitude 45°, $g = 980.6$ c.g.s.)
1 British thermal unit (1 pound of water raised 1° F. at 59° F.) = 778.8 foot-pounds at sea-level, latitude 45°.
1 gram-calorie (1 gm. of water raised 1° C. at 15° C.) = 4.190×10^7 ergs.
1 Joule = 10^7 ergs.
= 0.2387 gram-calorie.

TABLE III.

DENSITIES OF VARIOUS SOLIDS AND LIQUIDS.

Aluminum	2.6	Nickel	8.9	Mercury at 0°	13.596
Brass	8.1—8.6	Platinum	21.5	Acetic Acid at 15°	1.053
Copper	8.5—8.9	Quartz	2.65	Alcohol "	0.794
Cork	0.2	Silver	10.5	Aniline "	1.023
German Silver	8.5	Steel, Cast	7.8	Benzene "	0.884
Glass	2.4—2.7	Sulphur	2.0	Carbon Bi-sulphide at 15°	1.270
" Flint	3.1—3.9	Tin	7.3		
Gold	19.3	Vulcanite	1.3	Chloroform at 15°	1.499
Ice	0.917	Wax	0.96	Ether "	0.720
Iron, Cast	7.1—7.6	Wood, Beech	0.7	Glycerine "	1.260
" Wrought	7.8	" Ebony	1.2	Nitrobenzene "	1.20
Ivory	1.9	" Oak	0.7	Olive Oil "	0.915
Lead	11.4	" Pine	0.5	Toluene "	0.885
Marble	2.7	Zinc	7.1	Turpentine "	0.87

TABLE IV.

DENSITY OF VARIOUS GASES

at 0° C. and 760 mm.

	Referred to Water.	Referred to Air.	Referred to Hydrogen.
Air	0.0012931	1.00000	14.445
Oxygen	0.0014291	1.1052	15.964
Nitrogen	0.0012544	0.9701	14.013
Hydrogen	0.00008952	0.06923	1.000
Carbon dioxide . . .	0.001965	1.520	21.95
Mixed gases from electrolysis of water	0.0005360	0.4145	5.987
Aqueous vapor . . .	0.000804	0.6218	8.982

TABLE V.

DENSITY OF DRY AIR.

Referred to Water at 4° C.

t.	700 mm.	710 mm.	720 mm.	730 mm.	740 mm.	750 mm.	760 mm.	770 mm.
10°	0.001149	0.001165	0.001181	0.001198	0.001214	0.001231	0.001247	0.001264
11	1145	1161	1177	1194	1210	1227	1243	1259
12	1141	1157	1173	1190	1206	1222	1238	1255
13	1137	1153	1169	1185	1202	1218	1234	1250
14	1133	1149	1165	1181	1198	1214	1230	1246
15	1129	1145	1161	1177	1193	1209	1226	1242
16	1125	1141	1157	1173	1189	1205	1221	1237
17	1121	1137	1153	1169	1185	1201	1217	1233
18	1117	1133	1149	1165	1181	1197	1213	1229
19	1113	1129	1145	1161	1177	1193	1209	1225
20	1110	1125	1141	1157	1173	1189	1205	1220
21	1106	1122	1137	1153	1169	1185	1200	1216
22	1102	1118	1133	1149	1165	1181	1196	1212
23	1098	1114	1130	1145	1161	1177	1192	1208
24	1095	1110	1126	1141	1157	1173	1188	1204
25	1091	1106	1122	1138	1153	1169	1184	1200

TABLE VI.

REDUCTION OF WEIGHINGS WITH BRASS WEIGHTS TO WEIGHT IN VACUO.

$$k = \sigma\left(\frac{1}{\delta} - \frac{1}{\Delta}\right)$$

The table gives the values of k in mgms. when $\sigma = 0.0012$ gm. and $\Delta = 8.4$.

If the body weighed has the density δ, and weighs w grams in air, wk mgms. must be added to reduce the weight to vacuo.

δ	k	δ	k	δ	k	δ	k
0.6	+1.86	1.6	+0.61	5.0	+0.097	12.0	—0.043
0.7	1.57	1.7	0.56	5.5	0.075	13.0	—0.050
0.8	1.36	1 8	0.52	6.0	0.057	14.0	—0.057
0.9	1.19	1.9	0.49	6.5	0.042	15.0	—0.063
1.0	1.06	2.0	0.458	7.0	0.029	16.0	—0.068
1.1	0.95	2.5	0.337	7.5	0.017	17.0	—0.072
1.2	0.86	3.0	0.257	8.0	+0.007	18.0	—0.076
1.3	0.78	3.5	0.200	9.0	—0.009	19.0	—0.080
1.4	0.71	4.0	0.157	10.0	—0.023	20 0	—0.083
1.5	0.66	4.5	0.124	11 0	—0.034	21.0	—0.086

TABLE VII.

SPECIFIC VOLUME OF WATER.

Temp.	Volume.	Increase per degree.	Temp.	Volume.	Increase per degree.
0°	1.0001	0.000025	50°	1.0120	0.00048
4	1.0000	0.00005	55	1.0144	0.00052
10	1.0003	0.00012	60	1.0170	0.00054
15	1.0009	0.00018	65	1.0197	0.00058
20	1.0018	0.00022	70	1.0226	0.00062
25	1.0029	0.00028	75	1.0257	0.00064
30	1.0043	0.00032	80	1.0289	0.00066
35	1.0059	0.00036	85	1.0322	0.00070
40	1.0077	0.00040	90	1.0357	0.00074
45	1.0097	0.00046	95	1.0394	0.00076
			100	1.0432	

TABLE VIII.

DENSITY OF WATER.

Temp.	Density.	Diff.	Temp.	Density.	Diff.	Temp.	Density.	Diff.
0°	0.99988	5	10°	0.99974	10	20°	0.99825	21
1	0.99993	4	11	0.99964	10	21	0.99804	22
2	0.99997	2	12	0.99954	12	22	0.99782	23
3	0.99999	1	13	0.99942	13	23	0.99759	24
4	1.00000	1	14	0.99929	15	24	0.99735	25
5	0.99999	2	15	0.99914	15	25	0.99710	26
6	0.99997	4	16	0.99899	17	26	0.99684	27
7	0.99993	5	17	0.99882	18	27	0.99657	28
8	0.99988	6	18	0.99864	19	28	0.99629	29
9	0.99982	8	19	0.99845	20	29	0.99600	

TABLE IX.

REDUCTION OF METRIC BAROMETER to 0° C.

Brass Scale correct at 0° C. All corrections are subtractive.

Attached Thermometer.	Barometric Height in millimeters.								
Temp. Cent.	720	730	740	750	760	770	780	790	800
15°	1.76	1.78	1.81	1.83	1.86	1.88	1.91	1.93	1.96
16	1.88	1.90	1.93	1.96	1.98	2.01	2.03	2.06	2.08
17	1.99	2.02	2.05	2.08	2.10	.13	.16	.18	.21
18	2.11	.14	.17	.20	.23	.26	.29	.31	.34
19	.23	.26	.29	.32	.35	.38	.41	.44	.47
20	2.34	2.38	2.41	2.44	2.47	2.51	2.54	2.57	2.60
21	.46	.50	.53	.56	.60	.63	.67	.70	.73
22	.58	.61	.65	.69	.72	.76	.79	.83	.86
23	.69	.73	.77	.81	.84	.88	.92	.96	.99
24	.81	.85	.89	.93	.97	3.01	3.05	3.09	3.13
25	.93	.97	3.01	3.05	3.09	.13	.17	.21	.37

EXAMPLE.—Observed reading 763.59 mm. at 20°.5.
Interpolated corrections at 20°.5 for 760 and 770 mm. are —2.54 and —2.57.
Hence the corrected reading is 763.59 — 2.55 = 761.04.

TABLE X.

REDUCTION OF BAROMETER IN INCHES TO 32° F

Brass Scale correct at 62° F.

Attached Thermometer.	Barometer height in inches.				
Temp. Fahr.	29.	29.5	30.	30.5	31.
60°	.082	.083	.085	.086	.087
61	.084	.086	.087	.089	.090
62	.087	.088	.090	.091	.093
63	.089	.091	.093	.094	.096
64	.092	.094	.095	.097	.098
65	.095	.096	.098	.100	.101
66	.097	.099	.101	.102	.104
67	.100	.102	.103	.105	.107
68	.102	.104	.106	.108	.109
69	.105	.107	.109	.110	.112
70	.108	.109	.111	.113	.115
71	.110	.112	.114	.116	.118
72	.113	.115	.117	.119	.120
73	.115	.117	.119	.121	.123
74	.118	.120	.122	.124	.126
75	.120	.122	.125	.127	.129
76	.123	.125	.127	.129	.131
77	.126	.128	.130	.132	.134
78	.128	.130	.133	.135	.137
79	.131	.133	.135	.137	.140

EXAMPLE.—Barometer reading	30.452, 68°.2.
Correction under 30.5 and opposite 68. is	—.108
Corrected reading	= 30.344

Interpolations closer than 0.001 in. are needless.

TABLE XI.

REDUCTION OF INCHES TO MILLIMETERS

for the ordinary range of the barometer.

Inches and Tenths.	Hundredths of an inch.									
	.00	.01	02	.03	.04	.05	.06	.07	.08	.09
29.3	744.22	744.47	744.73	744.98	745.24	745.49	745.74	746.00	746.25	746.51
29.4	746.76	747 01	747.27	747.52	747.78	748.03	748.28	748.54	748.79	749.05
29.5	749.30	749.55	749.81	750.06	750.32	750.57	750.82	751.08	751.33	751.59
29.6	751.84	752.09	752.35	752.60	752.86	753.11	753.36	753.62	753.87	754.13
29.7	754.38	754.63	754.89	755.14	755.40	755.65	755.90	756.16	756.41	756.67
29.8	756.92	757.17	757.43	757.68	757.94	758.19	758.44	758.70	758.95	759.21
29.9	759.46	759.71	759.97	760.22	760.48	760.73	760.98	761.24	761.49	761.75
30.0	762.00	762.25	762.51	762.76	763.02	763.27	763.52	763.78	764.03	764.29
30.1	764.54	764.79	765.05	765.30	765.56	765.81	766.06	766.32	766.57	766.83
30.2	767.08	767.33	767.59	767.84	768.10	768.35	768.60	768.86	769.11	769.37
30.3	769.62	769 87	770.13	770.38	770.64	770.89	771.14	771.40	771.65	771.91
30.4	772.16	772.41	772.67	772.92	773.18	773.43	773.68	773.94	774.19	774.45
30.5	774.70	774.95	775.21	775.46	775.72	775.97	776.22	776.48	776.73	776.99
30.6	777.24	777.49	777.75	778.00	778.26	778.51	778.76	779.02	779.27	779.52
30.7	779.28	780.03	780.29	780.54	780.80	781.05	781.30	781.56	781.81	782.07
30.8	782.32	782.57	782.83	783.08	783.34	783.59	783.84	784.10	784.35	784.61

INTERPOLATION FOR THOUSANDTHS.

in.	mm.	in.	mm.
.000	.00	.005	.13
1	.03	6	.15
2	.05	7	.18
3	.08	8	.20
4	.10	9	.23

TABLE XII.

REDUCTION OF BAROMETER TO LATITUDE 45°.

CORRECTIONS ARE SUBTRACTIVE FROM 0° TO 45°, ADDITIVE FROM 45° TO 90°.

Lat.	720	730	740	750	760	770	780	790	800	Lat.
35°	0.64	0.65	0.66	0.66	0.67	0.68	0.69	0.70	0.71	55°
40°	.32	.33	.33	.34	.34	.35	.35	.35	.36	50°
Boston 42° 21	.17	.18	.18	.18	.18	.18	.19	.19	.19	—
45°	0.	0.	0.	0.	0.	0	0.	0.	0.	45°

Note.—These corrections make the allowance necessary for the variation in weight of the mercury with the latitude of the place of observation. The corrected reading gives the reading which would have been observed if the value of g had been the same as at 45°. It is indifferent whether this correction be applied before or after the reduction to 0° C.

If the barometer is above the sea level, a further slight correction for the diminution of g with altitude may be necessary. This correction is subtractive. but is only 0.015 mm. per 100 meters elevation. (See below.)

Example.—Observed barometer reading at Rogers Laboratory, 25 ft. above sea level, 753.28 mm. Corrected reading = 753.28 — 0.18 — 0 = 753.10.

FORMULA FOR VALUE OF g.

$$g = g_{0{\cdot}45}(1 - 0.00\,259 \cos 2\lambda)\,(1 - 0.00\,000\,020\,H),$$

where $g_{0{\cdot}45}$ = acceleration due to weight at sea level, latitude 45°;

$= 980.6\frac{cm}{sec^2}$, approx. (values vary widely in the next place of figures)

λ = latitude of place of observation.

H = altitude in meters above sea level.

TABLE XIII.

TENSION OF MERCURY VAPOR

IN MM. OF MERCURY.

(Regnault, Hagen, Hertz, Ramsay and Young.)

	mm.		mm.		mm.
0°	0.01	140°	1.9	280°	156
20	0.02	160	4.4	300	244
40	0.03	180	9.2	320	371
60	0.06	200	18.3	340	548
80	0.10	220	34.	360	790
100	0.30	240	59.		
120	0.80	260	97.		

TABLE XIV.

CAPILLARY DEPRESSION OF MERCURY IN A GLASS TUBE.

Interpolated from observations of Mendelejeff and Gutkowsky.

Diameter of Tube.	Observed height of meniscus in mm.							
	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8
mm.	mm.	mm.	mm.	mm.	mm.	mm.	mm	mm.
4	0.83	1.22	1.54	1.98	2.37	..	..	..
5	0.47	0.65	0.86	1.19	1.45	1.80	..	..
6	0.27	0.41	0.56	0.78	0.98	1.21	1.43	..
7	0.18	0.28	0.40	0.53	0.67	0.82	0.97	1.13
8	..	0.20	0.29	0.38	0.46	0.56	0.65	0.77
9	..	0.15	0.21	0.28	0.33	0.40	0.46	0.52
10	..	..	0.15	0.20	0.25	0.29	0.33	0.37
11	..	..	0.10	0.14	0.18	0.21	0.24	0.27
12	..	..	0.07	0.10	0.13	0.15	0.18	0.19
13	..	..	0.04	0.07	0.10	0.12	0.13	0.14

TABLE XV.

BOILING POINT *t*, OF WATER AT BAROMETER PRESSURE *H*.

Height of mercury column reduced to 0° C., 45° latitude, and sea level.

H	*t*	*H*	*t*	*H*	*t*	*H*	*t*	*H*	*t*
mm.	degrees	mm.	degrees	mm.	degrees	mm.	degrees	mm.	degrees
680	96.92	700	97.72	720	98.50	740	99.26	760	100.00
81	96.96	01	.76	21	.54	41	.30	61	.04
82	97.00	02	.80	22	.57	42	.33	62	.07
83	.05	03	.84	23	.61	43	.37	63	.11
84	.09	04	.88	24	.65	44	.41	64	.15
85	.13	05	.92	25	.69	45	.44	65	.18
86	.17	06	.96	26	.73	46	.48	66	.22
87	.21	07	97.99	27	.77	47	.52	67	.26
88	.25	08	98.03	28	.80	48	.56	68	.29
89	.29	09	.07	29	.84	49	.59	69	.33
690	.32	710	.11	730	.88	750	.63	770	.36
91	.36	11	.15	31	.92	51	.67	71	.40
92	.40	12	.19	32	.96	52	.71	72	.44
93	.44	13	.23	33	98.99	53	.74	73	.47
94	.48	14	.27	34	99.03	54	.78	74	.51
95	.52	15	.31	35	.07	55	.82	75	.55
96	.56	16	.34	36	.11	56	.85	76	.58
97	.60	17	.38	37	.14	57	.89	77	.62
98	.64	18	.42	38	.18	58	.93	78	.65
699	.68	19	.46	39	.22	59	.96	79	.69
700	97.72	720	98.50	740	99.26	760	100.00	780	100.73

PART II—OPTICS.

CONTENTS.

PAGE

Focal Length and Curvature of Lenses 5
Photometry 7
  General Discussion 7
Gas Photometry 15
Emission Spectra 20
  General Discussion: Flame, Spark, and Vacuum Tube Spectra 20
Chemical Spectroscope and Spectrum Analysis 25
The Spectrometer 30
  General Discussion of Adjustments 30
  Measurement of Angle of a Prism 34
  Index of Refraction of a Prism 36
  Index of Refraction of a Liquid 37
The Microscope 38

FOCAL LENGTH AND CURVATURE OF LENSES.

Object of Experiment.—The object of this experiment is to determine the focal length of a lens (*a*), optically, and (*b*), from measurements of its curvature, and incidentally to verify the law of lenses.

Apparatus.—The apparatus for the first part of the experiment consists of an optical bench or bar carrying a scale 400 cm. long, graduated in millimeters. At one end, *A*, a luminous "object," such as the filament of an incandescent lamp, is so placed that the plane of the filament is at the zero graduation of the scale. At the other end is a movable white screen, *B*, on which is focussed the image of the filament produced by the lens. The lens is mounted on a carriage, *C*, which can be adjusted at any intermediate position on the bar. The lens is provided with a set of diaphragms.

For the second part of the experiment a spherometer, millimeter scale, and glass plate are provided for measuring the curvature of the lens.

Procedure.—*First, to study the law of lenses and determine the focal length optically.*

Starting with the lens near the end of the bar farthest from *A*, set it at positions such that its distance, *AC*, from the light source or "object" is successively 320, 240, 180, 140, 120, 100, 80 cm., etc., these distances being chosen simply for convenience in subsequent calculations. Adjust the screen, *B*, until the image of the object is sharply defined, and record the position of *B* and of *C* in each case to the nearest millimeter. The screen should be adjusted so that the edge of the image is as nearly yellow as may be. It will be found that, to obtain a sharp image with the large lenses used, they must be "stopped down" with a diaphragm to reduce the effect of spherical aberration. Note the effect of the different types of diaphragm provided, and also the marked chromatic aberration of the lens on the color of the image. Note the difference in the focal length of the lens for different colored lights by placing a red and a violet screen in the path of the light beam.

As the lens approaches the object to such a distance that $AC = CB = 2f$, = twice the focal length of the lens, the intervals AC should be taken smaller and smaller. When $AC < 2f$, the interval should be decreased only a few centimeters at a time until the screen B moves off to the end of the bar.

Computation.—The law connecting the focal length, f, of a lens with its conjugate focal distances, p and p', is

$$\frac{1}{p}+\frac{1}{p'}=\frac{1}{f}. \tag{1}$$

A plot with values of p and p' as ordinates and abscissæ respectively will lead to a curve of the second degree, which can, however, be easily rectified. Let

$$\frac{1}{p}=\frac{1}{AC}=y \text{ and } \frac{1}{p'}=\frac{1}{CB}=x.$$

Equation (1) then becomes

$$y=-x+\frac{1}{f}=-x+f',$$

which is the equation of a straight line, making an angle of 45° with the axes of X and of Y respectively, and intercepting these at distances equal to the value of $f'=\frac{1}{f}$. Test the applicability of formula (1) to the data obtained, by means of a reciprocal plot and from its intercept on the axis of X and of Y respectively determine the mean value of f.

Second, determination of Focal Length from Radii of Curvature. Determine the radius of curvature of each side of the lens used in the preceding experiment by means of a spherometer as described on page 19, "Notes on Mechanics." Three or four settings on each side of the lens and on the plane surface will be sufficient. If n is the index of refraction of the glass of which the lens is constructed and R and R' are the radii of curvature of the two sides of the lens respectively, then

$$\frac{1}{f} = (n - 1)\left(\frac{1}{R} + \frac{1}{R'}\right). \tag{2}$$

The value of n for sodium light is marked on each lens.

Compute f and compare with the value obtained by the graphical method.

Questions and Problems.—1. How many significant figures do the data on p and p' warrant keeping in plotting their reciprocals?

2. What is the approximate precision of f, (*a*), as determined from the plot, (*b*), as computed from the curvature of the lens?

3. Suppose for a given plano-convex lens $f = 50$ cm. and $n = 1.6$, these being approximate measurements, and it is desired to determine f to 1 per cent.: (*a*), if the lens is so adjusted that it stands at an equal distance between the light source and its image, how precisely must the distance AB from object to image be measured? (*b*), how precisely must n and R be known if f is computed by formula (2)?

PHOTOMETRY.

GENERAL DISCUSSION.

The following discussion should be carefully read prior to beginning experimental work in Photometry. For further information on this subject Stine's "Photometrical Measurements" may be consulted.

Photometry is that special division of Optics which deals with the measurement of the intensity of sources of light and of diffuse illumination so far as this affects the eye. It is not concerned with the measurement of the intensity of radiations which lie beyond the limits of the visible spectrum. In ordinary photometry the integral effect of all visible radiations is measured: in spectro-photometry the distribution of intensity of the various colors of the spectrum is determined. A photometric measurement consists in general in comparing the intensity of the light to be measured with that of a standard by means of some form of photometer, of which there are numerous types. All consist, however, of a device by means of which a position can be found equally illuminated by the two light sources, or which will indicate when the intensity of these has been adjusted to equality. The measurement itself depends upon the judgment of the eye, and therefore rests upon a physiological and to a certain extent psychological basis. The eye, moreover, is a very poor judge of the intensity of a light source, the sensation not being proportional to the intensity of the light producing it. It has been shown that for an eye of normal sensitiveness the smallest variation in the intensity of a light which the eye is capable of detecting is about one per cent. of the total intensity, and this holds over quite wide ranges of intensity. Thus for an illumination of 10-candle power the eye can detect a change of 0.1-candle power, while for an illumination of 100 candles it cannot detect a change of less than 1-candle power. The following relation known as Fechner's law has been found experimentally to hold approximately between the intensity of a source and its physiological effect: *differences in sensations vary as the logarithm of the ratio of the stimuli producing the different sensations.* From the above it follows that in general the precision of a photometric measurement does not exceed about one per cent. Such measurements are therefore among the least precise, although by no means the least difficult, which are met with in physical work.

Standards of Light.—Of all standards of physical measurement the light standard is one of the least satisfactory because of the difficulty of accurately reproducing it. At the present time no single standard has been agreed upon. The following standards are those most frequently used or which have been proposed:—

1. *The Amyl-acetate or Hefner-Alteneck Lamp,*—used as a standard in Germany and now pretty generally adopted

as the standard for all scientific work. This consists of a wick lamp of the form shown in vertical section in Figure 1 and in perspective in Figure 2, and burning chemically pure amyl-acetate, a liquid which can readily be obtained of great purity. The wick tube, *A*, is 8 mm. inside and 8.3 outside diameter and 25 mm. high. The flame is adjusted to a height of 40 mm. A Krüss optical flame gauge, *B*, indicates when the proper height is reached. These lamps can now be obtained on the market, and may be *standardized* at the Reichsanstalt at Charlottenburg or at the National Bureau of Standards at Washington. A detailed description of this standard is given in the *Zeitschrift für Instrumentenkunde 13*, p. 257, 1893, also in Stine's "Photometric Measurements," p. 145. To emit one "Hefner unit," the lamp should burn under normal atmospheric pressure and average humidity (8.8 litres of moisture per cubic meter of dry air), and normal average carbon dioxide content, the flame being 40 mm. high. This standard has received the indorsement of the Committee on Units and Standards of the American Institute of Electrical Engineers, provided the lamp has been verified at the Reichsanstalt.

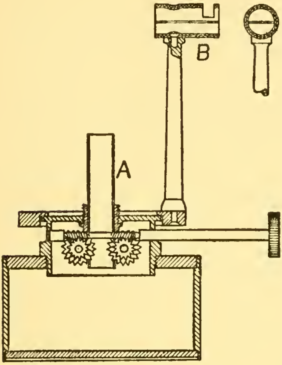

Fig. 1.

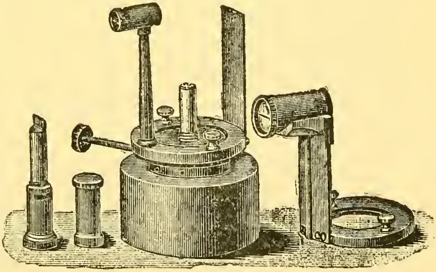

Fig. 2.

The disadvantages of this standard are its reddish color and the fact that the intensity varies slightly with the humidity of the air, the per cent. of carbon dioxide in the air, and the barometric pressure. The effect of each of these factors has been carefully studied, however, and in very accurate work may be corrected for.

2. *The Standard Sperm Candle,*—legally adopted as the standard in Great Britain and the United States, although a committee on standards of light of the British Association reported in 1888, "The present standard candle is not worthy in the present state of science of being called a standard." As defined, a standard candle must be made of spermaceti melting between 112° F. and 115° F. It must be 10 inches long, 0.9 inch diameter at the bottom, and 0.8 inch diameter at the top, and weigh one-sixth of a pound. The wick must be made of cotton, three strands of eighteen threads each being plaited together. A unit candle power is defined as the light emitted by a spermaceti candle conforming to the above-prescribed conditions of composition and construction when burning in dry air under normal atmospheric conditions with a flame 45 millimeters high and consuming 120 grains of combustible per hour. Its chief disadvantages are: uncertainty in the composition of the spermaceti; variation in weaving of the wick; effect of length and form of wick on the flame; reddish color of flame; effect of moisture in the air.

In Germany a standard *paraffin* candle (Vereins Normal Kerze) manufactured according to exact specifications, when burning with a flame of 50 mm. high, was generally used as a standard until the advent of the Hefner lamp.

3. *The Harcourt Pentane Lamp.*—In this lamp the vapor of pentane, a highly volatile hydrocarbon, is burned in a lamp of special construction, and the light emitted from the central section of the flame is taken as a standard. The quality of the pentane flame is excellent; but, owing to the difficulty of isolating pentane from other volatile paraffins associated with it, a chemically pure product is very difficult to obtain. The height and quality of the flame also depends on the tem-

perature of the vapor and air supply which the construction of the lamp renders difficult to control. This standard has, however, some excellent features to recommend it.

4. *The Carcel Lamp,*—originally designed by Regnault and Dumas, and used as a standard in France. This is a modified argand burner in which colza oil is burned. Although the flame is relatively white, as a result of the complete combustion produced by the central draft of the burner, it cannot be regarded as a satisfactory standard owing to wide fluctuations due to charring of the wick, the indefinite chemical composition of the oil, and variations in the mechanical construction of the lamp.

5. *The Methven Screen.*—This standard proposed by Methven (1878) was based on the assumption that, when gas is burned with complete combustion in similar argand burners, the light emitted per unit area from a certain region of the flame is constant, even though the gas varies in composition. He therefore constructed a screen with a rectangular slot of such an area that, when placed before the proper portion of a gas flame burning in an argand burner, the light received was equal to just 2-candle power. This standard came into quite extensive use both in England and America, particularly in gas-works; but subsequent careful experiments proved that the assumption upon which the constancy of the standard rested—namely, that the intensity of the flame is independent of the quality of the gas—was not valid. Further experiments to obviate this difficulty have since led to its practical abandonment as a standard.

6. *Violle's Platinum Standard,*—the light emitted normally by a square centimeter of molten platinum at its solidifying temperature. Investigation has shown this to be an impracticable standard owing to the cost of platinum, effect of impurities on its melting-point, and the experience and great skill in manipulation necessary to obtain reliable results.

7. *Lummer and Kurlbaum's Platinum Standard,*—now in use at the Reichsanstalt. This is defined as the light emitted

by one square centimeter of solid platinum heated by the passage of an electric current to such a temperature that 10 per cent. of the total radiation which it emits—measured by a bolometer—can pass through a layer of water two centimeters thick when contained in a cell with quartz faces. This standard requires elaborate apparatus and great skill on the part of the experimenter, and is adapted only for scientific research.

8. *Secondary Standards.*—In addition to the standards described above, secondary or comparison standards (*i.e.*, constant light sources which have been compared with some one of the preceding standards) are now in common use, particularly in connection with the photometry of arc and incandescent lights. By far the most important of these are incandescent lamps properly "aged" and run under specified condition of voltage. These give very satisfactory reference standards, being constant for long periods of time, and entirely independent of variations in atmospheric conditions, drafts, etc. The general subject of electric light photometry will be discussed at length later in connection with work in the Laboratory of Electrical Engineering.

Comparison of Various Standards.—The following values of the intensity of the more important of the above-mentioned standards in terms of Hefner units are the result of investigations undertaken by the German Gas Commission and German Physical Institute:—

1 English candle (flame 45 mm. high) = 1.14 Hefner units.
1 paraffin candle (Vereins Kerze, flame 50 mm. high) = 1.2 " "
1 pentane lamp set with 1 candle gauge = 1.17 " "

The value of the carcel lamp as determined by Laporte, 1898, is 10.9 Hefner units.

Photometers.—Of the many forms of photometer which have been devised, two of the most practical and widely used are the Bunsen disk and the Lummer-Brodhun screen.

The Bunsen Photometer.—This consists of a sheet of unglazed white paper at the centre of which is a translucent spot in the form of a circle or star, produced by paraffin, stearine, or other similar material. This should have a very clearly defined outline, to secure which it may be made as follows (Stine). Select a medium weight, white and smooth (but not glazed) linen bond paper for the screen. Then cut out a circle, star, or other design for the spot from a piece of sheet brass, and attach a handle to one side. Heat the brass in a Bunsen flame until a bit of paraffin or stearine placed upon it melts and runs off. When the film remaining is about to solidify, press it upon the centre of the photometer sheet. Then place a sheet of blotting paper over the screen and press with a hot iron, the heat from which will cause the paraffin to strike into the screen and produce a sharply defined "grease spot."

Another method of constructing a Bunsen disk is to cut out the design of the spot from two pieces of rather thin white paper and press between them one or more sheets of white tissue paper, the latter forming the translucent spot.

The Bunsen screen is usually mounted on a carriage in a box as shown in horizontal section in Figure 3. The two sides, S_1S_2, of the disk are viewed simultaneously through the opening O by means of two mirrors, M_1M_2, so placed as to reflect the surfaces of S_1S_2 to the observer. In making a measurement, the disk should be set so that the "spot" and its surroundings appear equally illuminated. No position can, however, be found where the spot appears to disappear completely when viewed simultaneously from both sides, owing to inequalities in the spectra of the lights and to the nature

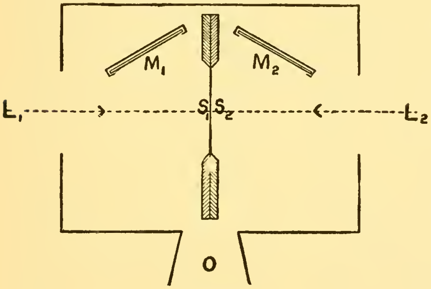

Fig. 3.

of the screen. The complete theory of the Bunsen screen is not simple. For an elementary discussion see Stine's "Photometrical Measurements," page 66. On the whole the best procedure is to set the disk at such a position that the contrast between the spot and its background appears the same when both sides are viewed simultaneously. For accurate work the screen and mirrors should be reversed to eliminate inequalities of the two sides.

Lummer-Brodhun Photometer.—The construction of this type of photometer is shown in Figure 4.

Light from the two sources, L_1L_2, falls normally on a white screen, S_1S_2, of calcium sulphate or magnesium oxide. The ingenious device for viewing simultaneously both sides of this screen was perfected by Lummer and Brodhun. M_1M_2 are two plane mirrors (or total reflecting prisms), symmetrically placed with respect to the screen, which reflect the light scattered at the two surfaces, S_1S_2, to the prism P, a device for viewing the two beams of light in juxtaposition. This is accomplished by constructing the prism in two parts with a common surface of contact, as shown in Figure 4. The rays, S_1M_1, fall normally upon the first face of the prism, and the central portion of the beam passes directly through to the eye-piece, T, where it appears as a circle of illumination. The outer portion of the beam incident beyond the surface of contact of the two prisms, is reflected back out of the field

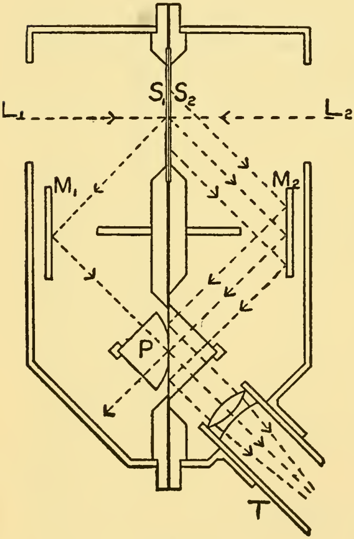

Fig. 4.

of view. On the other hand, of the rays of light, S_2M_2, coming from the other side of the screen, the central portion of the beam passes on through the surface of contact of the prisms and so out of the field of the eye-piece, while the outer portions are totally reflected into the eye-piece, appearing as an annulus surrounding the circle of illumination coming from S_1. As the contact between the two fields is exceedingly sharp, the adjustment for equality of illumination like S_1 and S_2 is greatly facilitated. Only when both lights are of the same quality can the distinction between annulus and circle be made to completely disappear. To eliminate the effect of inequalities in the surfaces S_1 and S_2, half of the settings should be made with S_1 presented to the light, L_1, and S_2 to L_2, and the other half with the screen reversed.

Observations with the Lummer-Brodhun screen are made with but one eye. Experience has shown that, for lights of similar color, settings can be made with greater precision than with the Bunsen disk. But the precision of setting distinctly diminishes after a number of settings have been made, owing to fatigue of the eye. When a large number of measurements are to be made, particularly with lights differing somewhat in color, a good Bunsen disk has been shown to be less fatiguing on the eye and, on the whole, to give the most satisfactory results.

GAS PHOTOMETRY.

Object.—The object of this experiment is to give practice in modern photometric methods, particularly as applied to the measurement of the intensity of gas flames and the efficiency of gas burners.

Apparatus.—The apparatus provided consists of a standard Hartmann & Braun photometer bar graduated in millimeters on the front scale and in "light ratios" (see below) on the back scale; a standard candle with candle balance and a Hefner lamp as light standards; a Bunsen and a Lummer-Brodhun photometer screen; also a wet gas meter for measuring the gas consumed and an alarm clock indicating minutes and seconds, to facilitate recording observations. Appropriate switches control electric lights for illuminating scales and measuring instruments when readings are to be made. The apparatus is mounted in special photometer rooms the walls of which are painted dead black.

Procedure.—*First;—to determine the ratio of the Hefner and candle units.* This involves adjusting the Hefner lamp so that the flame is burning under normal conditions, determining the rate at which the candle is consumed and measuring the relative intensity of the two lights.

Set up the Hefner amyl-acetate lamp at one end of the photometer bar and the candle balance at the other. Allow the lamp and standard candle (use in this test one candle only), to burn at least ten minutes before beginning measurements. Adjust in the mean time the amyl-acetate and candle flames to their proper height, see pp. 9, 10. When burning properly, the candle wick should curve at right angles and the end glow with a red heat. When using two candles, the plane of curvature of one wick should be at right angles to the other. The glowing end of the wick should be at right angles to the line of photometer, and in the vertical plane passing through the zero (or last) division of the photometer bar. When once the wick is formed, care should be taken not to break it; when lighting up, a drop of melted sperm should be dropped into the cup at the base of the wick, to prevent charring the wick before the cup begins to supply sperm. Upon extinguishing the candle, a drop of sperm should be allowed to fall on the glowing end immediately after the flame is blown out. When burning normally, the cup should be nearly dry and not full of melted sperm.

To determine the amount of candle consumed, proceed as follows: With the movable rider, *R*, placed on zero, adjust the counterpoise, *W*, of candle balance (see Figure 5) until it a little less than balances the candle; set the beam oscillating and observe the swing of the index on the scale, *S*, as the candle gradually burns away. Note by a watch or clock and record the time to the *nearest second*, when the candle is just balanced. No further attention need be paid to the candle during the next five minutes, during which time the disk settings are to be made. As the end of five minutes approaches, move the sliding rider, *R*, along 10 grains. If the candle has been burning at approximately its normal rate of 120 grains per hour (10 grains per five minutes), the candle side of the balance should now be a little too heavy. Again set the beam oscillating and note the time in seconds when the candle has just burned away enough to again bring the index to equilibrium. The difference between this and the initial time gives in seconds the time the candle has burned ten grains, from which its rate of consumption per hour may be calculated. If the rate is found to be between about 114 and 126 grains per hour, the corresponding candle power may be assumed proportional to the rate of consumption. If the candle is burning at a rate much greater or less than this, the wick should be attended to and a new series of observations taken.

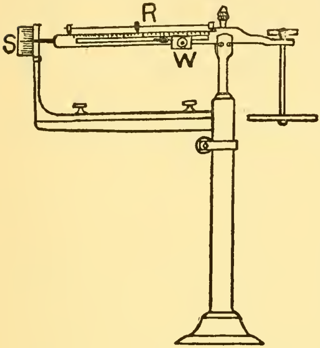

Fig. 5.

The relative intensity of the two flames is to be determined with the *Lummer-Brodhun photometer* screen. Take at least eight settings, half of them with the screen reversed to eliminate differences in its surface. Note that on account of the difference in quality of the two flames the field of the photometer cannot be made to appear absolutely uniform in any position. The disk should be set at that position where the spot and annulus appear as nearly equally illuminated as possible.

The photometer bar has two scales graduated upon it. The front scale is in linear centimeters and millimeters. The back or "light ratio" scale is so graduated that, when two lights, L_1 and L_2, are placed at the first (zero) and last (300 cm.) divisions of the bar respectively and the disk set for perfect equality of illumination, the reading, r, on the ratio scale corresponding to the reading, a, on the linear scale gives at once the ratio of the intensity of the two lights; *i.e.*,

$$r = \frac{L_1}{L_2} = \frac{a^2}{(300-a)^2}.$$

In this experiment the readings a should be recorded and r computed, in order to avoid errors in interpolating values on the unequally spaced divisions of the ratio scale.

Second;—to determine the efficiency of a given gas burner under different rates of consumption of gas. For a given type of burner the luminosity of a gas flame depends upon the rate at which the gas is consumed and upon the nature of the gas. For a given burner and kind of gas there is a rate of consumption at which the maximum light per cubic foot of gas consumed is obtained. Below and above this rate the gas is not burned most economically, although the burner may be giving out a larger amount of light. By the efficiency of a burner will be understood the number of light units which it emits per cubic foot of gas passed through it. To measure its efficiency, we need to know not only its intensity, but also the rate of consumption of gas. This is obtained by passing the gas which goes to the burner through a wet gas meter, Figure 6. For a

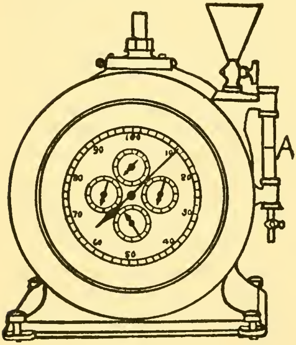

Fig. 6.

description of the internal construction of wet and dry gas meters, see Hornby's "Gas Manufacture." In the meter provided, when the water is adjusted to stand at the reference mark in the gauge *A*, one complete revolution of the long pointer indicates that one-tenth of a cubic foot of gas has passed through the meter. As the outer circle is graduated in 100 parts, thousandths of a cubic foot can be read directly, and 0.0001 cu. ft. by estimation. The small hands serve to count up the total revolutions.

The efficiency of the burner provided is to be measured in terms of Hefner units for at least five different rates of burning of the gas. The *Bunsen* photometer should be used for making the settings. At least four settings should be taken on each gas flame, two moving the disk from right to left and two from left to right. The gas should be so regulated that the lowest flame is about the size of a candle flame, and the highest, that with the gas turned on full force. The rate of consumption may be determined by recording the meter reading to thousandths of a cubic foot at a noted second before the measurements are begun and again at a noted second immediately after the disk settings are completed. To facilitate this procedure, a clock beating seconds is provided, which rings an alarm bell seven seconds before each minute, and a gong on the minute. If the meter is read each minute on the stroke of the gong, the differences of successive readings will give the rate of consumption per minute, and these should be constant if the gas is burning uniformly. From the average of these taken during a given series of disk settings the average rate per hour is to be computed.

The law in Massachusetts at the present time requires that illuminating gas shall give an illuminating effect equivalent to 16 standard sperm candles when burned at the rate of 5 cubic feet per hour in a burner commercially obtainable and best adapted to the purpose.

Computation and Report.—*a.* Compute the value of one candle power in Hefner units together with its average deviation.

b. Compute the value of one Hefner unit in terms of the standard candle and its average deviation.

c. Construct an efficiency plot for the burner studied, with efficiencies as ordinates and rates of gas consumption as abscissæ. From this plot determine the candle power of the burner when burning at its maximum efficiency and also when burning at the rate of 5 cubic feet per hour. Does the gas conform to the Massachusetts law, assuming the burner used to be the best type obtainable?

Problem.—The efficiency of a burner as determined in this experiment is given by the formula

$$E = r.\left(\frac{C}{120}\right).\left(\frac{1}{g}\right),$$

where $r = \dfrac{L_1 \text{ (burner at 0)}}{L_2 \text{ (standard at 300)}} = \dfrac{a^2}{(300 - a)^2}$,

a being the disk setting; C, the number of grains of candle burned per hour; and g the number of cubic feet of gas consumed per hour.

Suppose that the mean of the disk settings is $a = 239.9$ cm., $A.D. = 0.6$ cm.; the candle burns exactly 10 grains in 4 minutes 48 seconds, with a possible error of 2 seconds in the timing; the gas consumed in exactly 10 minutes is 0.8351 cu. ft., $A.D. = 0.0042$ cu. ft.

Compute the efficiency E and its average and percentage deviation.

EMISSION SPECTRA.

GENERAL DISCUSSION.

The best compilation and critical review of all matters pertaining to spectroscopy is given in Kayser's monumental work, "Handbuch der Spectroscopie," 4 vols.

Other more elementary works in English are Baly's Spectroscopy and Landauer's Spectrum Analysis.

Emission spectra are of three types,—continuous, banded, and line spectra.

Continuous Spectra.—These are characteristic of incandescent solids and liquids, the extent and intensity of the spectrum depending upon the temperature to which the substance is heated. The faint continuous spectrum often observed in flame spectra is in many cases due to the light emitted from the incandescent platinum used for introducing the salt.

The total energy, E, radiated from a hot body at absolute temperature, T_1, to surroundings at absolute temperature, T_2, can be shown to be expressed by the equation

$$E = \text{const.}\ (T_1^4 - T_2^4).$$

This relation is known as Stefan's law. The distribution of energy of various wave lengths throughout the spectrum of a hot body has been extensively investigated in recent years by Wien, Thiesen, Rayleigh, Planck, and others. The general form of the relation is

$$E = \text{const.}\ \lambda^{-5} f(\lambda T),$$

the form of the function f varying somewhat according to the results of different investigators. Several of the best methods of modern optical pyrometry are based on these laws.

Banded Spectra.—These are characteristic of some metals, but particularly of compounds. They often present striking peculiarities. In certain cases they are sharply and intensely defined on one edge and fade away towards the other, as in the case of barium. In others, as in strontium compounds, they have a maximum intensity at the centre and fade away on both sides. The spectra of the halogen salts of mercury offer excellent examples of banded spectra.

According to the kinetic theory banded spectra are, generally speaking, due to the vibrations of the molecules of a compound as a whole; line spectra (see below) are due to the vibratory motions of the elementary atoms. For this reason banded spectra can only be observed at comparatively low temperatures at which the compound is still undissociated. This is accomplished by using low-temperature flames or an electric discharge in a vacuum tube. As the temperature of the Bunsen flame is sufficient to dissociate many salts (particularly the nitrates and chlorides of the alkali and alkali earth metals), into their elements, the flame spectra of these compounds give us information chiefly regarding the metallic elements present.

Line Spectra.—These consist of sharp, well-defined bright lines separated by dark intervals, and are characteristic of gases and certain vapors. Substances may be brought into the gaseous and vapor state under conditions suitable for emitting their spectra by various modes of excitation, such as introducing them into flames, an electric arc, or spark, or, if gases at ordinary temperatures, into a vacuum tube through which an electric discharge is passed.

Flame Spectra.—For comparatively low-temperature spectra the substance is introduced into the outer portion of a Bunsen flame by means of platinum gauze attached to the end of platinum wire, or sometimes by soaking a bit of pure carbon with a solution of the salt. For somewhat higher temperatures the oxyhydrogen flame may be used.

Arc Spectra.—To obtain satisfactory spectra of the heavy metals, a much higher temperature than that obtainable in the Bunsen flame is required. The electric arc or spark is, therefore, usually employed for this purpose.

To produce an arc spectrum, the substance whose spectrum is to be studied is fed into the arc (between carbon electrodes) either directly or through the hollow core of one of the electrodes. A 110 volt direct current circuit connected through an adjustable rheostat and a 10 ampere adjustable or automatic arc lamp gives a brilliant arc well adapted for photographic or visual purposes.

In addition to the lines due to the vapors of the substance introduced, the arc spectrum contains the spectrum of carbon, or carbon compounds, and lines due to the many traces of impurities which it is almost impossible to eliminate from the electrodes.

Spark Spectra.—These are produced by observing the spectrum of a spark from an induction coil or static machine when passed between electrodes of the metal to be studied. In order to obtain sufficient brilliancy, a large *capacity* should be connected in *parallel* across the sparking terminals. This is conveniently provided in the form of Leyden jars.

In addition to the bright lines due to the metal volatilized from the electrodes, the spark spectrum also contains many air lines due to the heated gas through which the discharge takes place. These can, by the introduction of sufficient inductance in the circuit, be largely eliminated. In studying arc and spark spectra, the arc or spark should not be placed close to the slit, but at some distance from it, and the image of the light source formed on the slit by means of a lens. This has the advantage that the spectrum of different parts of the light source can be studied.

Vacuum Tube Spectra.—Gases and vapors at ordinary temperatures are rendered luminous for spectrum observations by enclosing them in a suitable form of vacuum tube, and passing an electric discharge through the tube. The form of vacuum tube best adapted for this purpose is shown in Figure 7, (*a*) and (*b*). These are so-called "end on" tubes, the discharge being viewed along instead of across the capillary, as in the ordinary form of Geissler tube, Figure 7 (*c*).

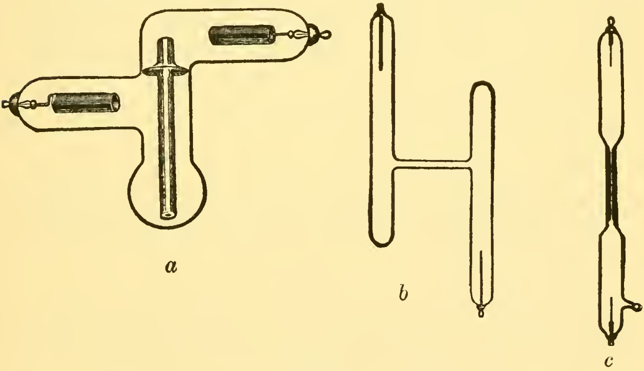

Fig. 7.

The intensity of the light source is thus greatly increased. It is not necessary to introduce a capacity in the circuit in parallel with these tubes, as in the case of the spark discharge. After a time, especially if driven hard, the vacuum "goes up" in most Geissler tubes, owing to the absorption of the gas by the electrodes. The discharge then passes with greater difficulty, and the Geissler effect gradually disappears, giving place, when a Crooke's vacuum is reached, to cathode rays, as shown by the phosphorescent effect which they excite on the glass. Such tubes are unsuited for spectrum work.

Spectrum Analysis.—The use of the spectroscope as one of the most delicate means of detecting the presence of an element has been recognized since the days of its invention. By it can be detected the presence, for example, of 0.0000003 mgr. lithium; 0.00002 mgr. thallium. It was, in fact, with the spectroscope that thallium and certain other rare elements were discovered. Its chief use has always been in qualitative rather than quantitative analysis. Attempts have been made to devise methods by which the amounts of substances present can be estimated, but, with a few exceptions, such attempts have led to no very satisfactory results.

The effect of temperature, pressure, magnetic field, and nature of excitation on the spectra of different elements, has been made the subject of very extensive investigations, and results of far-reaching interpretation have been obtained.

The explanation of the Fraunhofer lines of the solar spectrum laid the foundation for the modern science of astrophysics. The wonderful achievements of recent years in the study of the chemical constitution, and to a certain extent the physical condition of the sun and stars, together with their motions through space, are based primarily on the information disclosed by the spectroscope.

CHEMICAL SPECTROSCOPE AND SPECTRUM ANALYSIS.

Object.—The object of this experiment is to familiarize the student with the general character of gaseous, spark, and vacuum tube spectra, with the use of a chemical spectroscope, and with the elementary principles of spectrum analysis.

Apparatus.—The apparatus provided consists of a good chemical spectroscope, and the necessary accessories for producing and maintaining flame spectra, vacuum tube spectra, and spark spectra. These include an induction coil, separate condenser or Leyden jars, a spark stand, vacuum tubes, etc.

Refracting spectroscopes suitable for this experiment may be either direct vision instruments (Lockyer, Janssen), or instruments in which the rays coming from the slit suffer deviation (Bunsen and Kirchhoff). Both types of instrument are provided, and the student may use either or both.

Direct Vision Spectroscope.—The essential parts of this instrument are shown in Figure 8. The slit, *S*, is fixed at the principal focus of the collimating lens, *C*, which renders the rays parallel before reaching the system of prisms. This consists of three crown and two flint glass prisms arranged alternately, their angles being so chosen that rays are dispersed without being deviated. The spectrum is observed through an eye-piece, *E*, which can be focussed by a rack and pinion adjustment, *D*. *S′* is a scale, the light from which, after being rendered parallel by the small colli-

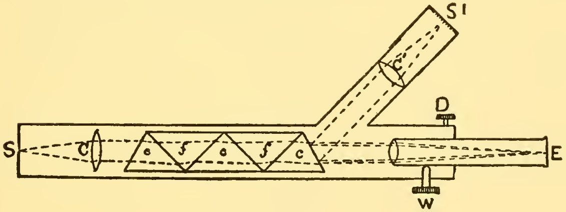

Fig. 8.

mating lens C', is reflected from the face of the last prism into the eye-piece. The adjusting screw, W, serves to move the eye-piece through the various regions of the spectrum.

Bunsen Spectroscope.—In this form of instrument only one prism, placed at its angle of minimum deviation, is used, whence the rays suffer a deviation, as shown in Figure 9. In other respects this instrument is similar to the preceding, the above description of the scale, eye-piece, etc., applying equally well to both.

Adjustment of Spectroscope.—Before observations with the instrument can be made, the collimator, scale and observing telescope must be adjusted for parallel rays. The following method of adjustment applies to either instrument. The adjustment of the collimator is usually made once for all when the instrument is constructed; *i.e.*, the slit, S, is placed at the principal focus of the collimating lens, C. To adjust the telescope, direct, by means of a mirror, a beam of strong daylight upon the slit along the axis of the collimator, or illuminate the slit by means of a sodium flame. Move the eye-piece back and forth until the Fraunhofer lines or

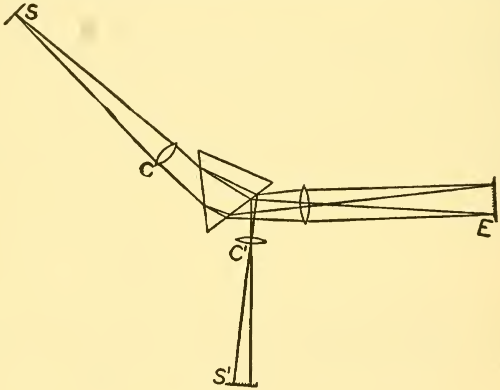

Fig. 9.

sodium lines are seen in sharp focus. In this adjustment the slit should be very narrow: the sodium lines, for example, as seen through the telescope, should not appear wider than half a scale division.

Having focussed the telescope, illuminate the scale, S', by daylight reflected from a mirror or by a gas flame, and adjust the tube carrying it, back and forth until it appears in focus. On now observing the sodium spectrum, the spectrum and scale should appear in good focus without parallax; *i.e.*, they should remain at rest with respect to each other when the eye is moved before the eye-piece. If this is not the case, the scale and spectrum do not come to a focus in the same plane, and the adjustment should be repeated.

Calibration of Scale.—Before a prism spectroscope can be used for quantitative work, the scale, which is an arbitrary one divided into equal parts, must be calibrated. This may be done by observing the scale reading of a number of well-defined lines whose wave length is known, and from these data constructing a plot with scale readings as abscissæ and wave lengths as ordinates. From this calibration plot the wave length of any observed line may then be found by looking out the ordinate corresponding to its scale reading. The only precaution to be observed in applying such a curve to subsequent observations is to see that at the time of observation the position of some one of the above lines coincides with its previously observed scale reading. If it does not, the scale has been displaced, and must be readjusted until such is the case.

The following table contains the wave lengths of a number of sharply defined lines which are well adapted for purposes of calibration. They are expressed both in microns μ (millionths of a meter) and also in ten millionths of a millimeter. These lines are all readily produced in a Bunsen flame.

With the dispersion of an ordinary chemical spectroscope, only the first three significant figures need be considered in the construction of the calibration plot.

TABLE OF WAVE LENGTHS.

Element.	Nature and color of line.	Wave length in microns μ.	Wave lengths in 10^{-7} mm.
Potassium	Red (double)	0.77019 0.76685	7,701.9 7,668.5
Lithium	Red	0 67082	6,708.2
Sodium	Orange (double)	0.58962 0.58902	5,896.2 5,890.2
Thallium	Green	0.53507	5,350.7
Strontium	Blue	0.46075	4,607.5
Rubidium	Violet (double)	0.42157 0.42020	4,215.7 4,202.0
Potassium	Violet (double)	0.40474 0.40443	4,047.4 4,044.3

Procedure.—*First.* Adjust the spectroscope as described above.

Second. Examine the spectra in the Bunsen flame of the various salt solutions provided. The spectra should be mapped directly on a sheet of 8″ x 10″ rectangular co-ordinate paper, as follows: Lay off on one side of the paper numbers corresponding to the scale divisions as seen in the eye-piece of the spectroscope. A sufficiently large scale should be chosen so that the whole visible spectrum from red to violet can be conveniently plotted without sacrificing any precision of the data. Having chosen the scale for plotting, map the spectrum of each salt by drawing in at their proper place the lines with their characteristic shadings, etc. Each spectrum should be one-half inch wide. Identify on the map the lines referred to in the table of wave lengths, which are to be used subsequently for drawing the calibration curve of the instrument.

The sodium line will probably be found present in every spectrum, owing to traces of sodium salts in the dust in the air or the several solutions. Only when it appears very prominent, should it be inferred to be present other than as an impurity. The violet potassium line is rather difficult to observe, except with a fairly wide slit.

The last solution to be examined is marked "unknown," and contains a mixture of several of the salts previously examined. The lines are to be mapped and identified and the contents of the solution thus determined.

Third. Examine and map the vacuum tube spectrum of hydrogen gas. Examine, but not record, the spectra of the several gases of the atmosphere,—oxygen, nitrogen, water vapor, argon, and also helium. Note the beautiful flutings in the nitrogen spectrum.

Fourth. Map the most prominent lines in the spark spectrum of magnesium by allowing the secondary of the induction coil to discharge across the magnesium terminals provided in the spark stand, and focussing sharply, by means of a lens, a horizontal image of the spark on the slit. The spectrum of a transverse section of the spark is thus obtained. Note the difference in length of the lines. Throw out the Leyden jar or condenser, and note the effect on the character of the spark.

Observe, but not record, the spark spectrum of copper and any other metals provided.

Fifth. Solar Spectrum. (Optional with the student.) Reflect strong daylight or sunlight (in which case the slit must be very narrow) on the slit parallel to the axis of the collimator. Record and identify by means of a table of spectra the principal Fraunhofer lines. From a comparison of the solar spectrum with the spectra previously observed what elements can you identify in the solar atmosphere?

Report.—This should include: 1. The original plot of spectra and on it the calibration curve. 2. Analysis of "unknown" solution. 3. Wave length of the principal hydrogen lines.

Questions.—1. Why, in the spectrum of a horizontal spark focussed across the slit, do some lines appear longer than others?

2. Why is it unnecessary to use a Leyden jar in circuit in the case of vacuum tube discharges?

3 Explain cause of different appearance of spark with and without Leyden jar in circuit.

THE SPECTROMETER.

INDEX OF REFRACTION OF SOLIDS.

Object.—The object of this experiment is to give practice in the adjustment of a spectrometer and in its use in determining indices of refraction.

Apparatus.—The essential features of a spectrometer or optical circle, the details in construction of which vary widely in instruments of different makers, are shown in Figure 10. These are: *first*, an accurately graduated circle, *A*, concentric with which is mounted, *second*, a movable plate or table, *B*; *third*, a collimator, consisting of a collimating lens, *C*, and a slit, *S*, adjustable in width; *fourth*, a telescope, *D*, provided with an eye-piece, *E*, for viewing the image of the slit. The telescope is so mounted as to move concentrically about the circle. The mounting is provided with a clamp screw, *G*, and a tangent screw to secure a fine adjustment of the position of the telescope. It also carries a pair of verniers 180 degrees apart. In high-grade instruments both telescope and collimator are provided with levelling screws, *F*, in addition to those on the base of the instrument itself.

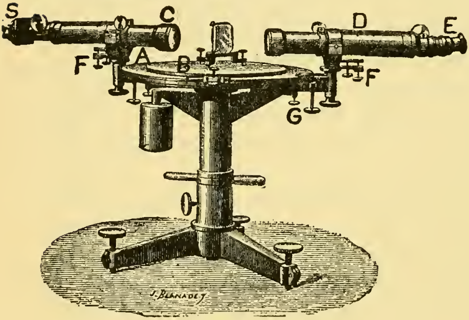

Fig 10.

Adjustments.—Before accurate measurements of angles can be made with an optical circle, the following adjustments of the instrument are necessary:—

1. The optical axes of the telescope and collimator must

be rendered perpendicular to and must intersect in the vertical axis of the instrument about which the prism table and telescope turn.

2. The telescope must be adjusted for parallel rays.
3. The collimator must be adjusted for parallel rays.

Procedure.—*First.—Adjustment of axes of telescope and collimator.* The optical axes of the collimator and telescope may, except in work of extreme precision,[1] be assumed to be parallel with the telescope and collimator tubes. If the telescope and collimator are not adjustable about their vertical axes, as is sometimes the case, it may be assumed that the maker has adjusted them so that their axes prolonged will intersect in the central vertical axis of rotation of the instrument. If, however, they can be adjusted, it should be seen that they are so directed that a pin placed vertically at the centre of the spectrometer plate is seen at the centre of the collimating lens when viewed through the slit, and at the centre of the telescope objective when viewed through the telescope with the eye-piece removed. Telescope and collimator should be firmly clamped once for all in this position.

To render the telescope perpendicular to the axis of rotation, turn it so that it is parallel to two of the levelling screws at the base of the instrument, and adjust them until the bubble of a striding spirit level placed on the telescope is at the centre. Now turn the telescope through 180 degrees, and, if the bubble is no longer at the centre of the level, bring it back to that position, making half of the adjustment by means of the levelling screws, *F*, on the telescope and half by the levelling screws at the base of the spectrometer. This adjustment must usually be repeated several times until the position of the bubble remains unchanged as the telescope is rotated.

To now adjust the axis of the collimator perpendicular to the central axis, attach a fine wire horizontally across the centre of the slit by means of a bit of wax; observe the image of the slit with the telescope, and adjust the collimator about its horizontal axis by means of its levelling screws, *F*, until the image of the wire across the slit coincides with the intersection of the cross wires in the eye-piece.

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[1] For method of adjustment see Watson's "Practical Physics," p. 290.

Second.—Adjustment of telescope and collimator for parallel rays.

First method,—applicable when a well-defined distant object several hundred feet or more away is available for focussing.

The eye-piece should first be focussed on the cross hairs with the eye in its natural unstrained condition. To insure this, turn the telescope towards a bright wall or object about two meters away, and throw it entirely out of focus so that the cross hairs are seen on an illuminated ground. Then with one eye at the telescope adjust the eye-piece until the cross hairs are seen distinctly at the same time that the other eye sees clearly the distant wall or object. If the observer is near-sighted and is observing without his glasses, it is better to direct the free eye upon a printed page held at the distance of most distinct vision in order to insure making the adjustment with the eyes unstrained. Having thus focussed the eye-piece on the cross wires, turn the telescope towards some object sufficiently distant (several hundred feet), to insure light rays coming from it being essentially parallel, and focus the eye-piece and cross hairs as a whole until the image produced by the telescope objective falls exactly on the cross hairs. They will then both be seen distinctly superimposed without parallax; *i.e.*, they will not shift relatively to each other when the eye is moved in front of the lens.

To focus the collimator when once the telescope has been adjusted, it is only necessary to direct the latter toward the former so that the image of the slit comes into the field of view, and then to adjust the slit back and forth until its image appears sharply defined on the cross hairs without parallax. The slit is then at the principal focus of the collimating lens the rays from which must issue parallel.

Second method,—applicable when no distant object is available upon which to focus the telescope for parallel rays.

Focus the eye-piece on the cross hairs as before. Then focus the telescope *approximately* for parallel rays by directing it upon some object in the room, as far away as possible. Next illuminate the slit with monochromatic light (sodium flame), and place a prism upon the table approximately in the position of minimum deviation, 1, Figure 11. Adjust the slit until its image as seen in the telescope appears sharp and well-defined. Now turn the prism so that the incident light falls more nearly normal upon the front face, as in position 2. It will be found that the telescope must now be turned so as to point in the direction of the arrow 2 in order to bring the image again on the cross hairs. Clamp the telescope in this position. The image of the slit will appear broader than before. If the edges are not sharply defined, refocus the *collimator.* Now turn the table upon which the prism rests back so that the prism passes through its position 1 (minimum deviation), and continue turning until a position 3 is reached, when the image is again seen on the intersection of the cross wires. It will now appear narrower than in the previous case, and, if not sharply defined, refocus the *telescope.* Continue this procedure of turning the prism alternately into position 2 and 3 and focussing the collimator on the broad and the telescope on the narrow image until both appear in good focus. This can evidently be the case only when both telescope and collimator are adjusted for parallel rays; *i.e.,* when the image of the slit comes to a focus at the same point in the telescope, independent of the length of the path traversed by the rays.

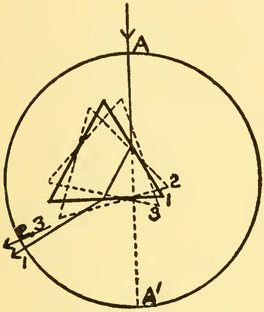

Fig. 11.

The instrument is now in adjustment and ready for use.

To Measure the Angle of a Prism.—*Adjustments.* To determine the angle between two faces of a prism, it must first be so adjusted on the spectrometer table that the plane of each face is parallel to the axis of rotation of the telescope. To facilitate this adjustment, the prism should be mounted on a triangular table provided with three levelling screws, and, for convenience in the adjustment, it should be so placed that one of the polished faces is perpendicular to the line connecting two of the levelling screws. See that this is the case before beginning the following adjustment. Place the prism with its centre over the centre of the spectrometer table, clamp the telescope at right angles to the collimator, and turn the table until the image of the slit reflected from the prism face is seen on the cross hairs of the telescope. Now cover up one-half of the slit by means of a shutter, if provided with the instrument, or, if not, stretch a bit of fine wire, as before, horizontally across the centre of the slit. Adjust the levelling screws of the prism table until the reflected image of the slit as seen in the telescope is brought to such a position that the centre of the slit (indicated by the edge of the shutter or by the horizontal wire) is at the intersection of the cross hairs. The face of the prism is then parallel to the axis of rotation of the telescope. Next turn the table until the image of the slit is reflected from the second face of the prism into the telescope, and again adjust the prism table until the centre of the image of the slit is at the intersection of the cross hairs. Care should be taken in this adjustment not to throw out the adjustment of the first face by turning the wrong levelling screw. It is to facilitate this adjustment that the prism should be mounted with one face perpendicular to the line connecting two levelling screws, for evidently in this position an adjustment of the third screw will move this face only in its own plane. A repetition of the above adjustment is usually desirable, after which one may proceed to measure the angle of the prism, as follows:—

First Method.—Prism fixed, and Telescope rotated. The usual method of determining the angle of a prism is to place the prism so that the angle to be measured is vertically over the centre of the spectrometer table and pointed towards the collimator. The beam of light coming from the latter is then partially reflected from each face. The prism is clamped in this position and the telescope moved and adjusted until the image of the slit reflected from the two faces falls successively exactly upon the cross hairs. The position of both verniers should be recorded in each position. The difference of their corrected readings gives twice the angle of the prism.

For a discussion of the error introduced if the angle of the prism is not over the centre of rotation of the telescope see Watson's "Practical Physics," p. 294.

Second Method.—Telescope fixed and Prism rotated,—applicable only when the spectrometer table is provided with verniers. In this case the centre of the prism should be placed over the centre of the table. The angle of the prism may then be measured by keeping the telescope fixed in position and turning the prism until the image of the slit, reflected successively from each face, falls exactly upon the cross hairs of the telescope. The angle of the prism is then evidently equal to 180 degrees minus the corrected angle through which the prism has been turned. For a discussion of the error introduced if the centre of the prism is not placed exactly over the centre of rotation of the spectrometer table see Watson's "Practical Physics," page 293.

Correction for Eccentricity.—The "eccentricity" of a circle is due to the non-coincidence of the centre of graduation of the circle with the axis around which the telescope revolves. Even in high-grade instruments, such as transits, goniometers, compasses, etc., it is practically impossible to obviate this defect in the mechanical construction of the instrument. If, therefore, only one vernier is attached to the instrument, the observed angular rotation will evidently not be equal to the true angular rotation by an amount depending upon the magnitude of the eccentricity.

By the use of two verniers, *A* and *B*, placed 180 degrees apart, it is easily shown by geometry that the error due to eccentricity may be eliminated by taking the arithmetical mean of the apparent angular rotations of the two verniers. Thus, if the readings of vernier A in the two positions are α_1 and α_2, and of vernier B, β_1 and β_2 respectively, then the true angular rotation of A from α_1 to α_2 is

$$\alpha = \tfrac{1}{2}\left[(\alpha_2 - \alpha_1) + (\beta_2 - \beta_1)\right].$$

If, however, either vernier, say A, moves through 0° or 360° in passing from α_1 to α_2, it should be noted that the observed angular rotation is not the difference of the observed readings, $\alpha_2 - \alpha_1$, but $\alpha_2 + (360 - \alpha_1)$.

If the line connecting the zeros of the two verniers does not pass through the centre of rotation, a constant error results; if the centre of graduation and centre of rotation do not coincide, a periodic error results. In general, a circle is subject to both sources of error, which are, however, eliminated by the use of two verniers, as described above.

To determine the Index of Refraction of a Prism.—The index of refraction of a prism for a given wave length λ is given by the formula:—

$$n_\lambda = \frac{\sin \tfrac{1}{2}(D_\lambda + \alpha)}{\sin \frac{\alpha}{2}},$$

where α is the angle of the prism and D_λ is the angle of minimum deviation of a ray of given wave length λ passing through the prism. To determine the index n_λ it is therefore necessary to measure besides α the angle D_λ for the wave length desired.

Measurement of the Angle of Minimum Deviation.—The angle of minimum deviation is the smallest angular deviation which a refracted ray can be made to undergo for any position of the prism; *i.e.*, it is the angle between the line of collimation AA', Figure 11, and the refracted ray 1, this being the position of the ray when it stops and begins to move back in the direction of the arrow 2, 3, as the prism is rotated through positions 2, 1, 3. The condition for minimum deviation is readily shown to be that in which the incident and refracted rays make the same angle with the first and second faces of the prism, respectively, or, in other words, that in which the path of the ray of light is symmetrical with respect to the prism. In practice the position is found by placing the prism on the table, as shown in Figure 11, and following with the *unaided eye* the position of the refracted image of the slit as the prism table is turned back and forth. The position of the prism for minimum deviation is that in which the image of the slit approaches nearest to the line of collimation. When this position is found, bring the telescope in line with the eye, when the image of the slit will appear in the field of view; clamp the telescope, and then with the tangent screw adjust until the intersection of the cross hairs exactly bisects the image of the slit, when the image reaches its position of minimum deviation. Record reading of both verniers.

Next find in the same way the corresponding position of minimum deviation on the other side of the line of collimation. The difference between the two readings of the telescope, corrected for eccentricity, in these two positions is evidently twice the angle of minimum deviation.

From this angle and the angle of the prism compute the index of refraction n_λ of the prism.

Index of Refraction of Liquids.—The index of refraction of a liquid may be determined in exactly the same manner as that of a glass prism, if a hollow prism with glass faces is available for containing the liquid. It is important that the plates forming the faces of the prism are plane parallel. As the index of a liquid varies considerably with the temperature, provision should be made for keeping the liquid at a constant known temperature or its temperature should be determined at the time of measurement. Other more convenient methods than the above have, however, been devised for measuring the refractive index of liquids, based on the laws of total reflection. One of the best of these is that by means of the Pulfrich Refractometer.

THE MICROSCOPE.

Object.—The object of this experiment is to give practice in the use of a good compound microscope, particularly in its application to micrometric measurements.

Apparatus.—The instrument provided is a compound microscope fitted with the following accessories: two objectives of approximately one inch and one-fifth inch focal length; two eye-pieces, one inch and two inches respectively, the former being provided with an adjustable eye-piece micrometer-scale rule in tenths of a millimeter; a substage equipped with an Abbé condenser and iris diaphragm; an Abbé camera lucida and drawing-table; and two Nicol prisms for observing objects by polarized light.

Description of Apparatus.—For the principle of the compound microscope consult the text-book on Physics. The path of the rays through the instrument is shown in Figure 12.[1] 1, 2, and 3 represent three pencils of parallel light coming from different parts of a distant source of illumination, such as a cloud. These are reflected by a plane mirror in the direction of the axis of the instrument. After passing through the condenser diaphragm, CD, the rays are collected by a condensing lens and focussed on the object at O_1. They then pass through the objective, the upper focal plane of which is at F_1. By the interposition of a collecting lens in the path of the rays the real image, O_2, of the object is transposed from F_2 to O_3 in the plane of the eye-piece diaphragm. When viewed through the eye-piece, this image is seen as a magnified virtual image, O_4, at a distance $C = 10$ inches, or 250 millimeters below the eye-point, EP, this being the distance of most distinct vision for the normal eye. The distance between F_1 and F_2 is called the optical tube length; the distance, L, is the mechanical tube length.

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[1] Figures 12 and 13 refer specifically to the Bausch and Lomb microscopes used in this experiment, and are reproduced through the courtesy of the Bausch and Lomb Optical Company.

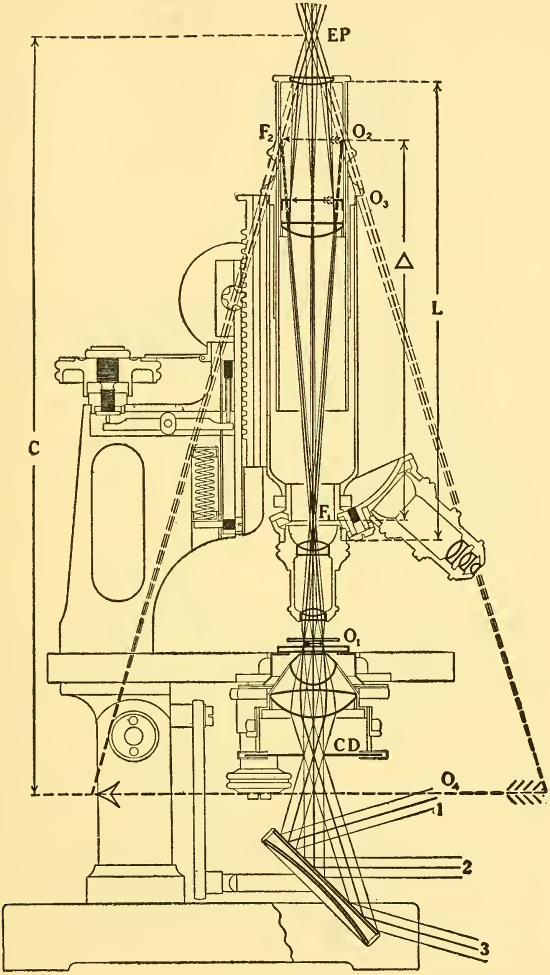

Fig. 12.

The various parts of a microscope, Figure 13, are designated as follows:

E = eye-piece.
D = draw-tube, for varying the magnification of the object. The graduations on the tube give the value in millimeters of the mechanical tube length, L, when nose-pieces are not used with the objectives.
T = body tube.
RN = revolving nose-piece for carrying several objectives, which may be revolved into place as desired.
O = objective in use.
R = rack for coarse adjustment.
PH = pinion head for focussing instrument approximately.
MH = milled head for fine adjustment of focus and, if graduated, for measuring the thickness of transparent objects.
HA = handle arm.
S = stage for carrying object under examination.
SS = substage for carrying diaphragm and condenser.
M = mirror for illuminating object from below.
B = base.
P = pillar.
I = inclination joint.

Illumination of the Object.—To obtain the best results with any microscope, care must be taken to secure the proper illumination of the object. If the object is semi-transparent, it should be viewed by transmitted light; if opaque, by reflected light concentrated upon it from above by an auxiliary condensing lens. To obtain the best results with transmitted light, the microscope should, if possible, be placed in a window having a north-sky exposure. Skylight is then reflected by means of the mirror, M, in the direction of the optic axis of the instrument. One side of the mirror is plane, the other concave. The latter is used to concentrate the light on the slide only when it is examined with a medium or a high-power objective. The plane mirror gives ample illumination with low powers. It is also always used in connection with the condenser. The intensity of the light should be so regulated that the field appears uniformly illuminated with a pleasant

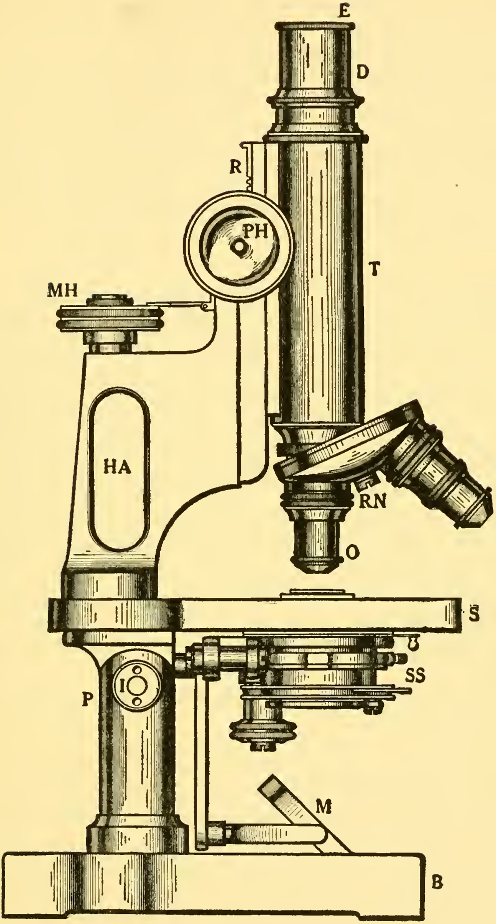

Fig. 13.

but not glaring light. If the illumination is too strong or too weak, the eye quickly becomes fatigued. Except for work with high-power objectives, the condenser and lower substage diaphragm should be swung aside, and the illumination regulated by the plane mirror and upper iris diaphragm. To obtain the best results with high-power objectives, however, the upper diaphragm should be opened to its fullest extent (to prevent injury by the end of the condenser), the condenser then swung back into place, and the light regulated by the lower iris diaphragm. A rack-and-pinion adjustment is provided for focussing the condenser.

Procedure.—*First. To become familiar with the general manipulation of the instrument.* The lowest magnification with the given equipment is obtained by using the low power, *i.e.*, long-focus objective, low-power (two-inch) eye-piece, and draw-tube pushed in. The highest magnification is obtained with the short-focus objective, short eye-piece, and draw-tube pulled out to the last division. By a combination of eyepieces and objectives and different positions of the draw-tube intermediate magnifications may be obtained. It is well, particularly for a beginner, to start with a low-power objective and work up to higher powers after the object has been found and carefully adjusted to the centre of the field of view. With high powers the field seen in the microscope is so very limited in area that much time and labor may be lost hunting for the desired object, if it is not first centred, as described above.

Observe the appearance of slide 1 under the following conditions:—

(*a*) Low-power objective, low-power eye-piece, and draw-tube in. Adjust the slide so that the object is at the centre of the field of view. Note the effect, other than changing the amount of illumination, of varying the size of the diaphragm.
(*b*) Note the effect of pulling out the draw-tube to its limit.
(*c*) Replace the low-power by the high-power eye-piece, and observe the increase in magnification.
(*d*) Replace the low-power by the high-power objective. To focus the instrument, lower the objective until it is nearly

in contact with the slide, and then, with the eye at the eye-piece, slowly raise the microscope by the coarse adjustment until the object comes into focus. This is the only safe procedure to follow, for, if the microscope is focussed down on to the object while the observer has his eye at the eye-piece, he may easily pass by the focus and crush the slide. All final adjustments of focus should be made with the fine adjusting screw, *MH*. In viewing the object under condition (*d*), the illumination should be regulated by the condenser and its diaphragm, to secure best definition. Note the effect of displacing the diaphragm laterally so as to illuminate the condenser obliquely.

Second. To calibrate the eye-piece micrometer for the lowest and highest magnification attainable with the two objectives provided. The shorter eye-piece is provided with an adjustable micrometer scale, graduated in tenths of a millimeter. When properly focussed, this scale appears superimposed upon the object seen in the microscope, and may be used to measure its linear dimensions if the value of one of the scale divisions in terms of the magnification employed is known. To determine this, the microscope is focussed upon a "stage micrometer" (slide 2), which consists of a glass scale ruled in divisions, 1/100 and 1/1,000 inch or 1/10 and 1/100 millimeter. From the number of divisions of the eye-piece micrometer coinciding with a known number of divisions of the stage micrometer, the value of one division of the former, for the particular combination of objective, eye-piece, and position of the draw-tube, is readily computed. Calibrate the eye-piece micrometer, first, with the low-power objective and draw-tube in, and, second, with the high-power objective and draw-tube drawn out to its last graduation.

Replace the stage micrometer by slide 3, and measure the diameter of the object (wire) under both magnifications, using the lower power first.

Third. Use of the camera lucida and determination of magnifying power.

The camera lucida is a device by which the image of the object as viewed in the microscope is seen projected outside the instrument on a drawing-table, where it may be traced out in detail by pencil or pen. The principle of the Abbé form of this device is shown in Figure 14. Over the eye-piece of the microscope is placed a small total reflecting prism, adjusted in optical contact over a small circular area at the centre of its reflecting surface with a second prism. Through this small area of contact the observer views the image of the object in the usual way. He also sees simultaneously, and apparently superimposed on the image, the point of a pencil placed on a drawing-table, *T*, at one side of the microscope, the rays from the pencil being brought to the eye by reflection from the 45° mirror, *M*, and the reflecting surface of the prism, *P*. In order that both object and paper may appear distinctly superimposed, they should be seen about equally illuminated. For this purpose the camera lucida is provided with two adjustable sets of graduated smoked-glass screens for varying the amount of light reaching the eye from the object and from the drawing-table, respectively. The camera lucida is readily attached to the eye-piece, as will be seen by inspection. An adjustable drawing-table with a rest for the arm and an attachment for holding the microscope is provided. After the microscope has been placed in position and focussed, the table should be so adjusted in height that the path of the rays reaching the eye from the paper is the normal distance of distinct vision, *i.e.* about 250 millimeters, or 10 inches.

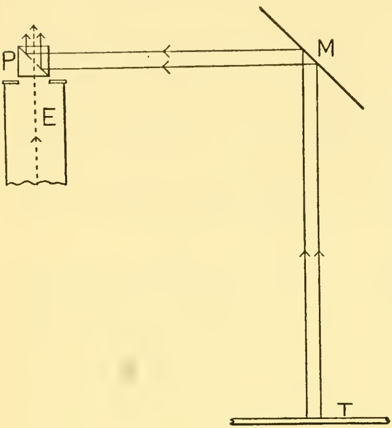

Fig. 14.

With the low-power objective and long eye-piece, and with the draw-tube in, make a drawing of the object on slide 4. Then replace the slide by the stage micrometer, and rule the lines as they are seen projected on the drawing. From the distance between these lines compute the linear magnification of the object.

Fourth. If time permits, it will be found interesting to observe several slides, containing doubly refracting salt crystals, in a beam of polarized light. The polarization apparatus will be furnished by the instructor upon request. The polarizer is a Nicol prism inserted in the path of the rays below the stage in place of the condensing lens. The analyzer, also a Nicol, is inserted between the objective and the eye-piece at the lower end of the body tube, T. The display of interference colors produced in this way, as the analyzer is rotated, is very beautiful.

A number of other interesting slides are provided for those who have time to examine them.

General Remarks. It is interesting to consider the limits which have been attained in the resolving power of the best microscopes. By "resolving power" is meant the ability of an instrument to separate two very near objects so they may be seen distinctly from each other. It can be shown that a theoretical limit of separation is imposed upon all instruments by the phenomena of diffraction, which, in turn, depends upon the wave length of light. The resolving power of a microscope objective thus depends upon the wave length of light used for forming the image. It also depends upon the index of refraction of the medium between the objective and object and upon the aperture of the objective. The "numerical aperture" of an objective, denoted by $N.\,A.$, is defined as $N.\,A. = n \sin \theta$, where n is the lowest refraction index between the object and front of the objective, and θ is one-half the angular aperture of the objective. If the illumination is such that the whole aperture of the objective is filled with light, the smallest distance that can be "resolved" is $d = \frac{\lambda}{2n \sin \theta}$ where λ is the wave length of light employed.

By the use of "oil immersion objectives" a very high resolving power is obtained, as n may be thereby increased from $n = 1.0$ for air to $n = 1.6$ for oil of cedar.

The reason for the particular advantage claimed for the recently developed "ultra-violet microscope" is also apparent from the above formula, since by using ultra-violet light as a source of illumination (in which case all optical parts of the microscope must be constructed of quartz instead of glass), the resolving power should be nearly doubled. Of course, with ultra-violet light the microscope cannot be used visually, but only for purposes of photography. As a matter of fact, owing to difficulties of focussing with invisible light, the definition obtained is not so good as could be desired, and the real distinctive advantage of this type of microscope is due to the unexpected selective absorption of micro-organisms for ultra-violet light. Thus, without staining the specimens, details of their structure are readily seen in the photographs. The size of the smallest objects which have been thus far actually measured is about 0.00001 inch.

Report.—This should contain:—

(*a*) Calibration of eye-piece micrometer for low-power objective, and draw-tube in.
(*b*) Calibration of eye-piece micrometer for high-power objectives, and draw-tube set on last division.
(*c*) Measurement of object under low and high magnification.
(*d*) Drawing of object with the camera lucida.
(*e*) Measurement of magnification of the drawing.
(*f*) Estimated precision of (*c*) and (*e*).

PART III—HEAT.

TABLE OF CONTENTS.

PAGE

THERMOMETRY: General Discussion 7
  Mercurial Thermometry — Part I. 14
  Mercurial Thermometry — Part II. 22
  Air Thermometry 27
  Pressure and Boiling Point 37
CALORIMETRY: General Discussion 42
  Specific Heat 55
  Latent Heat 61
  Mechanical Equivalent I. 67
  Mechanical Equivalent II. (Continuous Calorimeter) 72
EXPANSION 77
APPENDIX, Tables 78

THERMOMETRY

GENERAL DISCUSSION

These notes should be carefully studied before performing the experiments on Mercurial or Air Thermometry.

General Methods.— Thermometry is the art and science of temperature measurement. Any property of a substance which varies in a well known manner with its temperature may be made the basis of a system of thermometric measurement, and we find extensively used at the present time methods based on the following changes produced in bodies when heated or cooled:

a. Change of volume:—gas, mercury, alcohol thermometers.
b. Change of electrical resistance:—platinum resistance thermometers.
c. Change of thermo-electric force:—thermo-electric pyrometers.
d. Change of viscosity:—effusion or transpiration pyrometers.

In addition to these methods should also be mentioned specific heat pyrometers and optical pyrometers. The indications of the latter are based on the intensity or quality of light emitted by bodies at high temperatures. The term pyrometry is usually employed when referring to methods applicable to temperatures above 300° C.

The particular property or method which is best adapted to any given case depends of course upon the problem in hand as well as upon the actual temperature to be measured. Modern engineering as well as pure science demands methods covering a range of temperature from that of solid hydrogen—257.2° C. (Dewar, 1901), only 15° above the absolute zero, to that of the electric furnace, over 2000° C.

Of the several instruments mentioned above, gas thermometers are applicable over the widest range of temperatures.

When filled with hydrogen, they may be used down to near its point of liquefaction (—252.5° C., Dewar, 1901) and up to the highest temperature which a porcelain bulb will withstand, about 1400° C. A nitrogen thermometer can be used at even a higher temperature in a platinum-iridium bulb. Hydrogen diffuses through such bulbs at high temperatures. Gas thermometers afford the most accurate, absolute method of thermometry which we possess, but require considerable skill in their manipulation when a high degree of accuracy is desired. Several convenient commercial forms of less precision for high temperature measurements have, however, been perfected.

Mercury thermometers are far more convenient for ordinary use, and are applicable at temperatures from —20° C. to 550° C. A high degree of perfection has now been attained in the construction of these instruments, due in a large measure to a careful scientific study of the nature of the glass best adapted to withstand high temperatures without change. Fused quartz has also been recently (1903) proposed as a suitable substance of which to construct thermometers. The utmost precision attainable with the best modern instruments is 0.003° from 0°–100°, 0.1° from 100°–200°, and 0.5° from 200°–550°. Below the proper range of mercury thermometers, which become unreliable considerably above the temperature at which mercury freezes, —39° C., thermometers of similar construction but filled with alcohol, or better, toluene, are often used.

Less convenient, as they require complicated electrical auxiliary apparatus, are the electrical resistance or platinum thermometers, applicable at all temperatures, from the very lowest up to about 800° C. With these instruments temperatures can be measured to 0.001° up to 100°; 0.01° from 100°–500°; and 0.1° from 500°–800°.

Another instrument which of late years has rapidly and deservedly come into very general use, is the thermo-electric pyrometer, available at all temperatures up to the melting point of platinum, about 1700° C. This instrument is sensitive to 0.01° at 400° and to 1° at 1700° C. With a suitable galvanometer it may be very conveniently adapted to commercial work.

It is to be especially noted that, although both platinum resistance and thermo-electric pyrometers may have the high precision specified above when used in conjunction with suitable galvanometers, the *accuracy* of their indications in absolute degrees depends fundamentally on the accuracy with which the temperature of the melting or boiling point of various substances such as naphthalene, benzophenone, sulphur, gold, silver, etc., which are used for calibrating the pyrometers, is known. Such fixed points have been determined with the hydrogen thermometer. The pyrometers mentioned are therefore essentially secondary instruments depending upon a calibration which ultimately goes back to the hydrogen standard.

High temperatures may also be more or less accurately determined by several types of optical pyrometers, transpiration pyrometers, specific heat pyrometers, and others.

The following notes include a discussion only of mercurial and air thermometry of ordinary precision. They apply to the calibration of mercury thermometers ranging up to 350° C., and correspond to a precision not greater than 0.05° C. between 0° and 100°, and less than this above 100°. For a discussion of the various methods of pyrometry mentioned above the student is referred to the Laboratory Notes on Heat Measurements by Prof. C. L. Norton, or to Le Chatelier's and Boudouard's "Mesure des Températures élevées" (English translation by George K. Burgess). For the calibration and preparation of the highest grade thermometers the student should consult Guillaume's standard treatise on "Thermométrie de Precision," and the publications of the Reichsanstalt and of the Bureau des Poids et Mésures International.

Units.— The fundamental scale of temperature measurement is based on Thomson's (Lord Kelvin's) absolute thermodynamic scale, which is independent of the nature of any thermometric substance. The unit of temperature on this scale is called a *degree*, and has been chosen for convenience to coincide with a degree of the centigrade scale as defined below. The absolute scale is very nearly realized in practice by either a constant volume or constant pressure hydrogen thermometer, for the deviations of this gas from the laws of a perfect (ideal) gas have been shown by Joule's and Thomson's porous plug experiment to be so small that they are probably not greater than the experimental errors inherent in the best thermometric work.

The hydrogen thermometer without correction has been adopted by the International Bureau as the practical, ultimate standard of thermometric measurement, and represents at the present time the absolute scale of temperature as nearly as it is known. Measurements of temperature by other instruments than the hydrogen thermometer, not excepting even air or nitrogen thermometers, should therefore, if their accuracy warrants, be reduced to the hydrogen scale.

The Celsius or centigrade scale of temperatures is that universally employed in all scientific work. A centigrade degree is $\frac{1}{100}$ the temperature interval between the temperature of melting ice under 760 mm. pressure and the boiling point of pure water at the same pressure, these two fixed points being denoted by 0° and 100°, respectively. The value of a degree on the absolute scale and on the centigrade scale is, as stated above, the same. The zero of the former is, however, 273° below that of the latter. The relation between the two scales is $T = t + 273$, where T and t are expressed in absolute and centigrade degrees respectively, and $273 = \frac{1}{0.00367}$, the reciprocal of the coefficient of expansion of hydrogen gas at constant pressure.

The Fahrenheit scale, still employed (unfortunately) in much engineering work, is so graduated that the temperature of melting ice under standard conditions falls at 32°, and the temperature of boiling water under standard conditions at 212°. One Fahrenheit degree is, therefore, $\frac{1}{180}$ of this temperature interval, from which it follows that one degree Fahrenheit equals $\frac{5}{9}$ of a degree centigrade.

Any temperature on the Fahrenheit scale is reduced to centigrade degrees by the expression

$$t^\circ\, C = \frac{5}{9}(t'^\circ\, F - 32^\circ),$$

or conversely

$$t'^\circ\, F = \frac{9}{5}\, t^\circ\, C + 32^\circ.$$

On the Continent the Réaumur scale is often employed in daily life but never for scientific work. On this scale the temperature of melting ice is taken as 0°, and of boiling water under standard conditions as 80°. Hence one degree Réaumur is equal to $\frac{5}{4}$ degrees centigrade.

Mercurial Thermometers.—The best type of mercurial thermometer for general scientific work is one with a cylindrical bulb of about the diameter of the stem and with the graduations etched directly on the glass (*a*, figure 1). The graduations should be equidistant and should represent as nearly as may be, degrees or some simple fraction thereof. The capillary should terminate in a small bulb at the top to facilitate separating a thread for calibration purposes, and also to prevent accident in case of over-heating. When the total range of the thermometer is limited to a few degrees, and it is desired to use it at widely different temperatures, as, for example, in boiling or freezing point determinations, the capillary is provided with a cistern at the top as shown in *b*, figure 1, by means of which the amount of mercury in the bulb can be adjusted and the thermometer made to read at any temperature. In certain cases when the thermometer is to be used only over a limited number of degrees at two different temperatures, as, for example, in calorimetric work in the neighborhood of 0° and at room temperature, the unused interval, (5°–15°), is removed by blowing a small bulb of proper volume in the capillary just above the 5° division (*c*, figure 1). This device avoids making such sensitive calorimeter thermometers of excessive length. In many thermometers of German make, the stem of the thermometer is a small capillary tube, back of which is placed a white porcelain, graduated scale and the whole inclosed in a protecting glass tube (*d*, figure 1). These thermometers are easy to read and are convenient for many purposes, but the scale is liable to move, the thermometers are

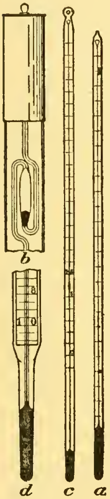

Fig. 1.

bulky, and for anything except relative measurements are less desirable than those described above.

Sources of Error.—The temperature indicated by a mercurial thermometer is subject to the following sources of error, which must always be investigated and corrected for in thermometry of even moderate precision, *i.e.*, where an accuracy of 0.1° C. or more is desired.

First.—Errors arising from irregularities in the diameter of the bore. The correction for this source of error is known as the "calibration correction."

Second.—Errors arising from the fact that often a portion of the stem of the thermometer is exposed to a different temperature from that of the bulb. The thermometer will, therefore, read too low or too high, according as the bulb is above or below the temperature of the stem. The correction for this error is known as the "stem exposure correction."

Third.—The error resulting from the so-called "lag" of the thermometer. The correction is referred to as the "lag ice reading." This often serious source of error requires further explanation. It arises from the fact that glass, after undergoing a sudden change of temperature, does not return immediately to its original volume when its initial temperature is re-established, but lags behind. Thus, if a thermometer has been heated to say 100°, and is then suddenly cooled to 0°, the volume of the bulb will remain temporarily too large and the ice reading will be too low. In the course of time the "zero" will gradually rise as the bulb contracts and returns to its original volume. The converse will of course be true if the bulb is strongly cooled and the temperature then raised.

The amount of "lag" depends on the composition of the glass, on the duration of heating or cooling, on the temperature to which the thermometer is heated, and on the rapidity with which it is cooled from this temperature to zero degrees. On rapid cooling (*e.g.*, in one or two minutes) from 100° to 0°, thermometers often show a lag from 0.1° to 0.5°, according to the kind of glass of which they are made. The time required for the lag of a thermometer to disappear, rapidly diminishes the higher the temperature. Thus the lag resulting from a sudden cooling of 100° would be several months in disappearing at 0° C.; a few days at 100° C., and only a few hours at 200° C. It is evident, therefore, that the lag observed in any case depends to some extent on the previous history of the thermometer as well as upon the immediate change of temperature.

As the bulb of a thermometer is necessarily very strongly heated during the process of construction and filling, the zero point of a new thermometer is often observed to rise continually, sometimes by several degrees. This change in the volume of the bulb is known as "ageing" of the thermometer. It may often be removed and always reduced in amount by a process of annealing, *i.e.*, heating the thermometer for several days or hours to a temperature above that at which it is to be used, and then allowing it to cool very slowly. Thermometers treated in this way are now prepared by some French makers; they are marked "récuit," and are usually much superior to others. The best thermometers are made at present either of "verre dur" in France, or of "Jena Glass" in Germany.

Fourth.—Errors due to the value of one scale unit not being exactly one degree. This arises from the fact that the ice and steam points are not exactly 100 scale units apart.

Fifth.—Errors resulting from the deviations of the thermometer from the absolute or hydrogen scale. The correction is known as the "reduction to the hydrogen scale."

In high grade thermometry, *i.e.*, where an accuracy greater than 0.05° is desired, corrections are also sometimes necessary for the external pressure on the bulb of the thermometer which may cause its zero to vary; the internal pressure of the mercury column which varies with the height of the column and the inclination of the thermometer; and for capillarity, which causes the indication of a thermometer to vary according as the mercury meniscus is ascending or descending. To eliminate the effect of internal pressure the International Bureau recommends that thermometers be read in a horizontal position.

MERCURIAL THERMOMETRY — PART I

CALIBRATION AND SCALE UNIT OF A THERMOMETER

Object.— The object of the experiment on Mercurial Thermometry is to give the student a practical working knowledge of the proper procedure in measuring temperature by means of a mercurial thermometer with an accuracy of 0.1° C. The experiment is divided into two distinct parts: Part I, the calibration of the thermometer and the determination of the value of its scale unit; and Part II, the use of a thermometer as illustrated by the measurement of a definite temperature—the boiling point of some liquid—with particular reference to the correction of all sources of error affecting the result is 0.1° C. or more.

Apparatus.— A thermometer will be assigned to each student, to which he should attach a tag bearing his name. At the end of the exercise thermometers are to be returned to an instructor for safekeeping until the next exercise. After reports on the *completed* experiment (including both Part I. and Part II.) have been handed in and *accepted*, the thermometers with their receipts are to be returned to an instructor, who will credit them to the student if found in as perfect condition as when delivered. The thermometers provided (*a*, figure 1) are graduated in centigrade degrees, and have a range from about — 12° to + 112° C. The graduations are equidistant and etched directly on the stem. The end of the capillary terminates in a small bulb blown at the top of the thermometer.

Parallax correctors in the form of brass tubes about one centimeter in diameter and ten centimeters long, and blackened on the inside, are mounted vertically in blocks for reading the mercury thread. Parallax may also be eliminated by placing the thermometer on a mirror and bringing the eye to such a position that the end of the thread and its image coincide. For very accurate calibrations short focus reading telescopes are provided.

Procedure.— *Calibration of a Thermometer.* The calibration consists in recording the length of a thread of mercury about ten degrees long, throughout the entire length of the capillary. Since the volume of the thread remains constant, a change in its length at any point of the capillary indicates an irregularity in the diameter of the bore at that place. From the variation in length of the thread throughout the tube, a complete plot of the form of the bore can be deduced.

The first operation consists in clearing the capillary of the mercury which partially fills it at ordinary temperatures. To do this, allow the mercury to run up into the small bulb at the top of the capillary until it is *partially* filled. Then, holding the thermometer in a nearly horizontal position, give the upper end a quick, light tap with a small block of soft pine wood. The thread will usually break and the mercury can then be separated into two parts, one partially filling the bulb at the top, and the other filling the true thermometer bulb, leaving the whole length of the capillary clear. Care should be taken not to allow the upper bulb to become completely filled with mercury, as under these circumstances it is often difficult to get the mercury down again into the bulb proper.

The next step is to break off a thread of suitable length for calibration from the mercury in the large bulb. This is a delicate operation, and an unskilled manipulator is likely to crack the thermometer at the point where the stem is joined to the bulb. Students should first observe an instructor separate a thread before undertaking to do it themselves. The procedure is as follows: Hold the middle of the thermometer in the fingers of the left hand and tap the upper end sharply with a small piece of soft wood until a thread of suitable length is ejected from the bulb into the capillary. Often several small threads of mercury can be separated and afterwards joined together to the proper length. Several trials are usually necessary.

Next carry the thread to the upper end of the stem by gently jarring the thermometer in an inclined position against the fingers or hand supporting the upper end. It is much safer to do this at the start and to work the thread *down* rather than *up* the stem during the calibration; for there is little danger of the mercury in the small bulb at the top running out and joining the thread, whereas with well boiled-out thermometers this is very likely to occur with the mercury in the large bulb if the thread is being worked *up* the capillary. The thermometer should be handled only at its extreme ends during all of the following manipulation in order to avoid local heating of the capillary which inevitably produces apparent irregularities. In the finest work the thermometer should not be handled at all during calibration, but manipulated from a distance and read by a telescope.

Starting with the thread at the extreme upper end of the thermometer, read the position of its upper and lower ends, a_1 and a_2, respectively, to 0.1° (a good observer can estimate to 0.05°). Record the observations in two vertical columns, leaving room for a third column for the values a_1–a_2, etc. Next jar the thread slightly so that it moves down the capillary a few tenths of a degree and record the position of its ends again, a'_1, a'_2. This second observation serves simply as a check on the first, for if the observations have been made with care the lengths a_1–a_2 and a'_1–a'_2 should agree, for it is quite improbable that the diameter of the capillary changes by an appreciable amount in the length of a fraction of a degree.

Next move the thread about one-half its length down the stem and record the position of both ends, checking these observations as before by a second observation with the thread a fraction of a degree further on in the capillary. It is undesirable that the end of the thread should fall under a graduation, as the accuracy of estimation is less in this position than in any other, on account of the width of the graduations. Proceed in this manner throughout the whole length of the capillary to below the zero graduation. Compute all the differences, a_1–a_2, etc., before joining the thread with the mercury in the bulb. If any large irregularities are found, carry the thread back to that part of the capillary and explore by intervals of say one or two degrees, instead of half the length of the thread, in order to locate exactly where the irregularity occurs.

The mercury in the top of the thermometer can usually be readily rejoined with that in the bulb by attaching a short, stout string through the small loop in the end of the thermometer and swinging the thermometer in a circle. The centrifugal force expels the mercury. This procedure is always effective, provided the upper bulb is not completely filled with mercury. If this is the case, it is often difficult to connect the mercury, as a minute quantity of air in the capillary will prevent a thread run up from the bulb from joining the mercury at the top. The mercury can sometimes be made to flow down into the bulb by jarring the thermometer, but more frequently it is necessary to expel a portion of it from the small bulb by heating the upper end of the thermometer above a flame. If the bulb can be thus partially emptied of its mercury contents and the remaining mercury allowed to contract in ice, this portion can then be ejected by whirling the thermometer as described above.

Determination of the Scale Unit.—To determine the value of this quantity the reading of the thermometer in steam, followed by its reading in ice, must be known.

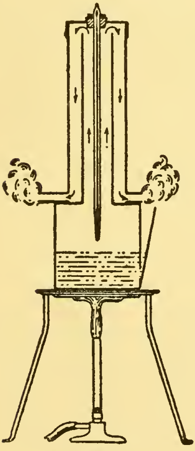

Fig. 2.

First. Steam Reading.—The apparatus for taking the steam reading is shown in figure 2. The thermometer is suspended by means of a section of a rubber stopper fitting it tightly several degrees above the 100° mark, which rests upon a perforated stopper closing the top of the heater. The whole thermometer should hang in the inner steam jacket of the boiler as shown, the bulb being an inch or two above the surface of the boiling water. The bulb should not be immersed in the boiling water itself for the following reason: If the water contains impurities, its boiling point is slightly raised, which would of course cause the "steam reading" to be too high. If, on the other hand, the bulb comes in contact only with the vapor rising from the water, the vapor condenses on the thermometer and forms a coating of pure water on the bulb, the temperature of which does not rise above the true boiling point of the pure water. The water should boil briskly, but not so violently as to spatter upon the bulb. Clean tap water may be used, although distilled water is preferable.

When the water has boiled for five minutes or more, raise the thermometer just high enough to read the mercury column, and record the temperature. Record the steam reading again several times and take the mean as the best representative value. The thermometer should then be carefully removed from the heater and allowed to cool in the air to about 40° C., when it should be plunged into the ice bath and the "lag ice reading" taken. The reading of the barometer should also be recorded, from which the true temperature of the steam may be found in the steam tables.

Second. The Lag Ice Reading.—While the water is coming to a boil in the preceding operation, prepare a can, of a liter or more capacity, *full* of very finely crushed ice. The ice should be crushed especially fine if it has been artificially prepared or the weather is very cold, as under these circumstances its temperature is likely to be considerably below zero. The ice must be clean and pure. Add enough ice water to nearly fill the can. The upper layer of ice should look white. For work to 0.1° C., tap water may be used. For exact work distilled water must be used. The bulb of the thermometer must be clean; the presence on it of any impurities soluble in water may lower the temperature in its immediate surroundings very appreciably. Make an opening in the ice for inserting the thermometer, with a clean glass rod.

When the temperature of the thermometer has fallen to 40° C., plunge the thermometer into the ice bath fully to the zero mark, and record readings every half minute for five minutes or more. The lowest reading is to be taken as the "lag ice reading." With thermometers reading to only 0.1°, the "lag" reading does not usually change appreciably during the first ten minutes or so that the thermometer is in ice. With more sensitive calorimeter thermometers the immediate rise of the zero is very evident.

This completes all data necessary for computing the scale unit.

Computation. — The computation of this experiment consists first, in the calculation of the calibration corrections of the thermometer and the construction of a plot of corrections, and second, in the calculation of the value of the scale unit.

First. Calibration Corrections.—Construct a plot with values of the length of the calibrating thread a_1–a_2, a'_1–a'_2, etc., as ordinates and corresponding values of the position of the *lower* end of the thread a_2, a'_2 etc., as abscissæ. The scale of ordinates should be such that the precision of plotting will not be greater than ten times that of the data. For abscissæ, a scale of $1'' = 10°$ is suggested. The best representative line should be a *smooth* curve passing among the points, and not a broken line. This curve will give a graphic representation of the irregularities of the capillary. The ordinate corresponding to any abscissa is the length which the calibrating thread would have had in the capillary if its lower end had been placed at the position corresponding to the abscissa in question. It is now desired to ascertain what any given reading of the thermometer would have been if the bore had been uniform throughout instead of irregular; in other words, what corrections must be applied to any observed reading in order to reduce it to the true reading of a perfect thermometer of uniform bore.

To determine this we first proceed to determine a series of consecutive "equal volume points" corresponding to the volume of the mercury thread chosen for calibration. Suppose we start with the lower end of the thread at any position A, near the lower end of the bore; its length in that position will be given by the ordinate y_1 of the calibration curve corresponding to the abscissa A. Its upper end will therefore read $A + y_1$. If this thread be now supposed moved along the tube until its lower end stands at $A + y_1$, its length will then be the value of the ordinate y_2, corresponding to the abscissa $A + y_1$ of the curve, and its upper end will therefore read $A + y_1 + y_2$. In this way we can determine the readings A, $A + y_1$, $A + y_1 + y_2$, etc., of a series of points which mark off, consecutively, equal volumes along the tube. If the capillary were perfectly uniform, these points would be equally spaced. Make a four column table. Record in the first column the reading A, which in this case should be taken a few degrees below zero. The zero itself is often a convenient starting point if no temperatures below zero are to be corrected for calibration. Record in column II the ordinate y_1 corresponding to A as read off to hundredths of a degree from the plot. Note that rejection, in the computation, of figures in the hundredths place may result in an accumulated error of one or more tenths of a degree. Record in this way a series of equal volume points to above the 100° division.

Let the last equal volume point nearest to and just above 100° be B, and suppose there are n equal volume intervals between A and B. The *average* length of the thread y_m between the points A and B will then be $y_m = \dfrac{B - A}{n}$.

I.	*II.*	*III.*	*IV.*
A	y_1	A	$A - A = 0$
$A + y_1$	y_2	$A + y_m$	$A + y_m - (A + y_1)$
$A + y_1 + y_2$	y_3	$A + 2y_m$	$A + 2y_m - (A + y_1 + y_2)$
...	...	...	...
...	...	...	...
...	y_n	...	...
$A + y_1 + \ldots + y_n = B$		$A + ny_m = B$	$B - B = 0$

If, therefore, the capillary had been uniform in cross section throughout, and we had started with a thread of the length y_m with its lower end at A, the equal volume points between A and B would have been A, $A + y_m$, $A + 2y_m$, etc., the last point falling at $A + ny_m = B$. Record these values in column III. The difference between the actual equal volume points, column I, and the calculated equal volume points for a perfect thermometer, column III, is due to the irregularities in the bore, and represents the corrections which must be added to the actual readings in order to reduce them to the true readings of a perfect thermometer. These corrections are to be recorded in column IV. The correction for any intermediate reading on the thermometer may be obtained by interpolation on a plot of corrections, which should be constructed with values in columns I and IV as abscissæ and ordinates, respectively. This curve may conveniently be drawn on the same sheet with the calibration plot.

Note that the numerical value of the corrections depends on the points which are selected for A and B, as their value determines the cross section of the imaginary uniform tube to which the corrections are referred. This makes no difference, however, in the value of the final reduced temperature, as this latter further involves the value of the scale unit (see next paragraph), which depends upon the ice and the steam reading, corrected for calibration *from this same plot*, and which varies, therefore, in a corresponding manner.

Second.—Value of the Scale Unit.

Let t'_s = mean steam reading corrected for calibration by preceding plot,
t'_i = lag ice reading corrected for calibration,
t° = actual temperature of steam at time steam reading was observed.

Then the mean value of one scale unit of the thermometer in degrees is

$$a = \frac{t^\circ}{t'_s - t'_i}.$$

The value of t° may be computed from the formula

$$t = 100^\circ + \tfrac{3}{80}(H - 760)$$

where H is the reduced barometric reading in millimeters. The value may also be found by interpolation from suitable tables. (See Appendix or Landolt and Börnstein, Physikalisch-Chemische Tabellen, p. 60).

MERCURIAL THERMOMETRY—PART II

USE OF A MERCURIAL THERMOMETER AND DETERMINATION OF THE BOILING POINT OF A LIQUID

Methods of Using a Thermometer.— In order to obtain the true temperature corresponding to any observed reading of a mercury thermometer, it is necessary that a definite procedure be followed in the manipulation, and that the several errors affecting the indications of the thermometer be corrected for in a definite sequence. There are two methods of using a thermometer which depend primarily on the determination of its zero or ice reading, which, as has been pointed out, is a constantly varying quantity. These methods will be referred to as the "ordinary" and "lag methods," respectively.

The Ordinary Method.— In the ordinary method the ice reading is taken immediately *before* the temperature which is to be determined; it also *precedes* the steam reading in the determination of the scale unit, p. 21. Little or no attention is paid to the lag except by expert observers. In other respects this method is the same as the lag method. It is liable to errors of 0.1° C. to 0.°5 C. between 0° and 100°, and is therefore not to be followed in accurate work.

The Lag Method.— In this method, which is the only one by which the full accuracy of which a thermometer is capable can be attained, the ice reading is taken in every case immediately *after* the temperature measurement. The "lag reading," *i.e.*, the lowest temperature observed when the thermometer is cooled rapidly from the observed temperature to zero degrees, is taken as the ice reading. It is sometimes called the "maximum depressed zero," or "depressed zero." For work accurate to 0.1° C. to 0.2° C., the lag reading may be taken some time before or after the actual temperature measurement itself, by heating the thermometer to about this temperature and cooling rapidly to zero. The elapsed time must not, however, be so great that the zero, meanwhile, may have changed by more than the experimental error. The law expressing the relations between the lag at 0°, and the temperature interval through which the instrument has been cooled, will be found in Guillaume's Thermométrie.

Stem Exposure.— One other precaution must always be observed in temperature measurement whenever the whole thermometer, or at least the whole mercury thread, is not exposed to the same temperature as the bulb. For if the bulb is above the temperature of the major portion of the mercury thread in the stem the thermometer will evidently read too low, and conversely. The correction is known as the *stem exposure correction*, and can be computed approximately if the number of degrees of mercury in the exposed stem and its temperature is known. The latter is determined by surrounding the exposed part of the thermometer with a water jacket, the temperature of which is determined by an auxiliary thermometer. A convenient arrangement for this purpose is a glass tube closed at the bottom by a perforated rubber stopper through which the thermometer passes, as shown in figure 3.

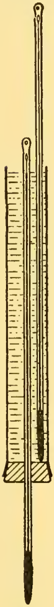

Fig. 3.

The stem exposure correction is computed as follows:

Let t' = observed apparent temperature of bulb corrected for calibration,

t_a = temperature of the stem as determined by auxiliary thermometer,

n = number of degrees exposed at temperature t_a,

0.000156 = coefficient of apparent expansion of mercury in glass,

t_b = true temperature of bulb (desired).

Then the reading of the thermometer corrected for stem exposure will be

$$t'' = t' + 0.000156\,(t_b - t_a)\,n.$$

As t_b is really unknown we use its approximate value t' without appreciable error, and write

$$t'' = t' + 0.000156\,(t' - t_a)\,n.$$

It is easy to see that at 100° the correction may amount to 1° C. It is a source of error too often neglected in work of even moderate precision.

Order of Application of Corrections. — Suppose that the thermometer used has been calibrated and a plot of corrections determined, and that the value of its scale unit is known. A temperature is subsequently determined as described above, the proper data for the exposed stem correction, as well as the lag ice reading, being observed. The true temperature corresponding to such data is then computed as follows, the corrections being applied in the order indicated:—

Let t = observed temperature of the thermometer,
t_a = observed temperature of exposed stem,
n = number of degrees exposed at temperature t_a,
t_i = lag ice reading,
a = value of one scale unit.

First.—Calibration Correction. Correct the observed temperature for calibration by means of the plot of calibration corrections. The correction taken with its proper sign should always be added.

$$t' = t + c.$$

Second.—Stem Exposure Correction.

$$t'' = t' + 0.000156\,(t' - t_a)\,n$$

Third.—Lag Ice Reading.

$$t''' = t'' - t_i.$$

Fourth.—Reduction to Degrees by Value of Scale Unit.

$$t^{\text{IV}} = a\,t'''.$$

Fifth.—Reduction to Hydrogen Scale. This last correction may be omitted, although if the kind of glass of which the thermometer is constructed is known, the correction may be readily obtained. To determine the correction experimentally a direct comparison of the mercury and hydrogen thermometer would be necessary.

The amount of this correction for various kinds of glass is given in the following table, taken from Guillaume's Thermométrie:—

ALGEBRAIC EXCESS OF READING ON SCALE SPECIFIED OVER THAT OF THE HYDROGEN THERMOMETER.

Scale.	10°	20°	30°	40°	50°	60°	70°	80°	90°
Nitrogen . . .	+0.006	+0.010	+0.011	+0.010	+0.009	+0.005	+0.001	−0.002	−0.003
Verre dur . .	+0.052	+0.085	+0.102	+0.107	+0.103	+0.090	+0.072	+0.050	+0.026
French crystal (hard) . . .	+0.064	+0.107	+0.130	+0.138	+0.134	+0.119	+0.097	+0.069	+0.036
French crystal (ordinary) . .	+0.067	+0.112	+0.137	+0.147	+0.144	+0.130	+0.107	+0.076	+0.041
Jena glass 16^{III}	+0.057	+0.093	+0.113	+0.119	+0.116	+0.102	+0.083	+0.058	+0.031
Jena glass 59^{III}	+0.024	+0.036	+0.037	+0.033	+0.026	+0.016	+0.007	+0.001	−0.002
English crystal (Wiebe) . . .	0.000	+0.02	+0.03	0.00	−0.03				

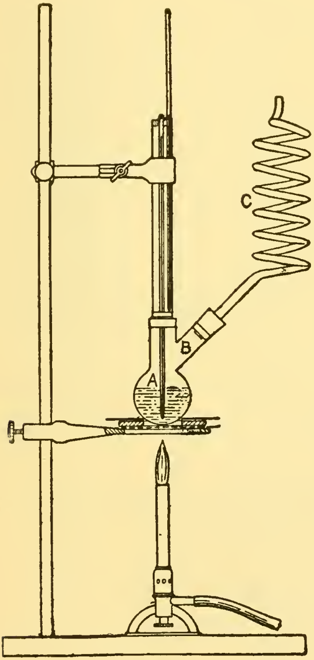

Fig. 4.

Object.—The object of the following experiment is to give practice in the proper method to follow in exact mercurial thermometry. The procedure is illustrated by the determination of the boiling point of some liquid.

Apparatus.— The student is to use the thermometer which he has calibrated. The apparatus required is shown in figure 4. *A* is a small boiling flask provided with a side tube *B*, which is connected with a return condenser *C*. The flask is clamped in position on an asbestos and wire gauze support, so constructed that the direct flame does not come in contact with the flask. The thermometer is held in place by a cork fitting the neck of the flask; the bulb should be wholly immersed in the liquid or in the vapor according as the boiling point of the former or temperature of the latter is to be determined. The addition of glass beads, garnets, or scraps of platinum, to the flask is frequently necessary to insure quiescent boiling. The necessary accessories for determining the stem exposure correction as described in Thermometry, Part I., are provided.

Procedure.—*First,* determine the boiling point of the pure liquid provided. Use the *short*-neck flask, arranging the apparatus as shown in the figure. Allow the liquid to boil for five minutes or more, during which time prepare an ice bath for the determination of the lag ice reading. At the end of five minutes record four or five readings of the thermometer and take their mean. Record also the temperature t_a of the water surrounding the exposed stem, as indicated by the auxiliary thermometer, and the number of degrees of mercury thread exposed at this temperature. The reading of the barometer and its attached thermometer should also be recorded. Finally, determine the lag ice reading, as described on page 18.

Second, determine the boiling point of the solution provided, using for this purpose the *long*-neck flask. See that the condenser is clean and dry. The solution must not be boiled violently, otherwise vapor will escape from the condenser and the boiling point will continually rise. Finally, raise the bulb of the thermometer well above the surface of the liquid and determine the temperature of the vapor.

Computation.—Compute the true boiling point of the pure liquid, the solution, and its vapor, stating the reduced barometric pressure at the time of the experiment.

Problems.—1. How many degrees of a thermometer may be exposed at a temperature of 20° C. with its bulb at 100° C., and the stem exposure correction be less than 0.1° C.?

2. What will be the error resulting from neglecting the stem exposure correction of a thermometer reading 300° C. with 200° of the stem exposed at temperature of 20° C.?

3. Explain the reason of the results obtained on the temperature of the solution and of its vapor.

AIR THERMOMETRY

Object.— This experiment illustrates the use of the air thermometer for temperature measurement or for the determination of the coefficient of expansion of a gas. The boiling point of water under atmospheric pressure is to be determined, assuming the value of the coefficient of expansion of the air in the thermometer known.

Apparatus.— The apparatus provided is a constant volume air thermometer consisting of a thin glass, spherical (or cylindrical) bulb *A* blown at the end of a capillary tube *B*, which is bent in the manner shown in figure 5. The other end of the capillary is connected to a glass tube *C*, about one centimeter in diameter which is sealed into a three way steel stop-cock *S*, by means of wax. This cock is connected by heavy rubber pressure tubing to a second glass tube *D*, of the same internal diameter as *C*. Both *C* and *D* are fixed to movable slides which can be clamped at any desired height along the vertical support *H*. The height of *D* may be further adjusted by means of a fine adjusting screw *E*. A scale graduated in milli-

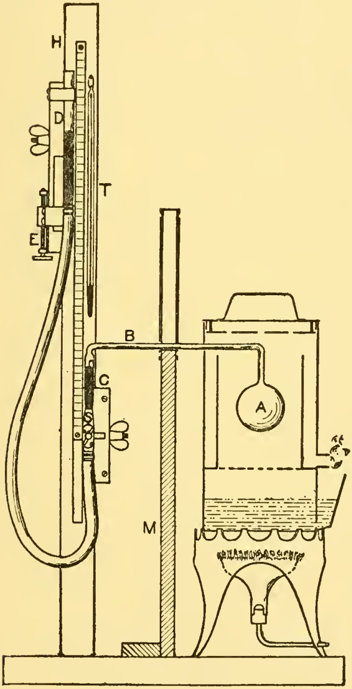

Fig. 5.

meters is attached to the front side of the support *H*. The height of the mercury column in *C* and *D* is read on this scale by means of a cathetometer or reading telescope.

The three way stop cock *S* requires further explanation. Its construction will be seen from figure 6. 1 is a vertical section; 2, a front view of the cock in the same position when it is closed. It is seen that the through boring is in the direction of the handle of the cock, and the lateral boring at right angles to this direction. This is indicated on the cock by the marked or flattened side. The only positions in which the cock is closed is when the handle stands at 45°, as in 1, 2 and 5. 3 is the proper position for putting *C* and *D* into free communication; 4 is the position for filling the thermometer with dry air, and for drawing off the mercury from *C*.

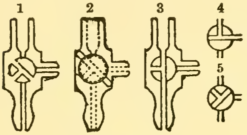

Fig. 6.

The boiler consists of a double walled vessel of copper in which the vapor of the boiling liquid circulates up around the bulb and down between the inner and outer walls, thus preventing direct radiation from the outside walls on to the bulb of the thermometer. A slotted screen *M* protects the mercury columns from the direct radiation of the boiler.

Procedure.—*First. To fill the thermometer with pure, dry air.* (This operation may be omitted if the instructor states that the apparatus is already properly filled.) Lower the bulb into the heater and bring the water to a boil. Lower *D* below the level of the mercury in *C*, and then open the stop-cock *S*, so that the mercury in *C* flows back into *D* until the horizontal opening of the stop-cock *S* is clear. Connect this with a T-tube, one end of which is attached by means of rubber tubing to the drying apparatus, and the other to an air pump, as in figure 7. Both of the rubber tubes should be provided with independent pinch cocks, P_1 and P_2. The drying apparatus should be so arranged that fresh air is drawn through a strong solution of caustic potash or soda lime, to remove carbonic acid, then through several dehydrating agents, such as calcium chloride or strong sulphuric acid, and finally over phosphorous pentoxide. While the water is boiling, pump the air from A through P_1, P_2 being closed. Then close P_1 and *gradually* open P_2, allowing dry air to be drawn into the apparatus. Close P_2 and open P_1. Repeat the above operation a dozen times or more. After the last filling allow the bulb to cool to room temperature before disconnecting the drying apparatus. Then turn the stopcock S, so that C communicates freely with D, and raise the latter until the mercury appears in C. The thermometer is now filled with dry air at approximately atmospheric pressure and room temperature.

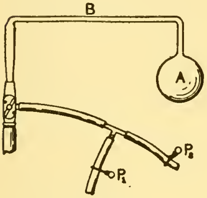

Fig. 7.

Second. To determine the pressure when the gas occupies a definite reference volume at a known temperature,—the melting point of ice.—This operation in gas thermometry corresponds to the determination of the zero point of a mercurial thermometer.

Crush enough ice to fill the inner cylinder of the heater. Cover the wire bottom of the cylinder with about an inch of ice; then lower the thermometer bulb into it until the capillary stem rests on the bottom of the slot in the side of the heater. During this operation be sure that S is closed, otherwise the mercury will run over into the bulb. If this should occur, speak to an instructor. Care must be taken to exert no pressure on the bulb, otherwise the thermometer is likely to snap off at C. Pack the bulb and stem of the thermometer with ice, filling the inner cylinder completely. Cover the top of the ice with a piece of felt or flannel to prevent circulation of air currents. Place the cover on the heater and allow the apparatus to stand at least ten minutes before taking any readings. A thermometer T should, in the meantime, be suspended along the vertical support H with its bulb close to C, in order to obtain the temperature of the air and mercury in C, both of which are sasumed to be at the same temperature.

While waiting for the air to cool to 0°, record the height of the barometer and its attached thermometer. After about five minutes open the stop-cock S cautiously, so that C and D freely communicate, and adjust the height of D so that the mercury in C just comes in contact with the point of the black glass reference mark. The adjustment should be made with a *rising* meniscus. If the mercury column now remains at rest for several minutes (the cock S being open), the air in the bulb may be assumed to have reached a constant temperature, and the height of the mercury columns in C and D should be recorded. This is to be done by means of a reading telescope provided with a horizontal cross hair. The field of view must include both the mercury meniscus and the graduated scale. Estimate readings to 0.1–0.05 mm. For very accurate work, (readings to 0.01–0.02 mm.), the eyepiece of the telescope must be provided with a filar micrometer, or a cathetometer must be used. Repeat the adjustment and observations two or three times and take the mean of the readings in computing the final pressure. Record the temperature t_1 of the thermometer T. As long as the mass of air in the bulb remains unchanged, the reduced pressure p_o, corresponding to this data, will be the same whenever the air is cooled to 0° C. and the volume is adjusted so that the mercury in C stands at the reference mark. For a given thermometer containing a constant amount of gas, this pressure p_o is a constant. It is always well, however, to determine the pressure corresponding to the ice reading prior to a temperature determination in order to eliminate possible errors arising from leakage or change of volume of the bulb.

Third. To measure an unknown temperature.—To measure any temperature with a given air thermometer for which p_o has been determined as above, it is only necessary to find the pressure which will bring the gas to the original reference volume when heated to the temperature in question. The method and procedure is to be illustrated by determining the boiling point of water (or any other liquid provided), under atmospheric pressure. Close the stop-cock S. Light the gas under the boiler, which should be filled with water to a depth of three or four centimeters. Tap water is sufficiently pure for this experiment, although for work of extreme accuracy, distilled water should be used. The ice need not be removed from the heater unless it is desired to save time, as the steam will quickly cause it to melt and run back into the boiler. While waiting for the ice to melt, again record the reading of the barometer and its attached thermometer. When the ice has melted and the water has boiled briskly for five minutes, raise D about twenty centimeters and *carefully* open S. Adjust the height of D until the mercury in C comes exactly to the reference mark. Wait several minutes to see if the pressure remains constant; if so, record the position of C and D as before. Readjust two or three times and take the mean of the readings. Record also the reading t_2 of the thermometer T. Finally close S, turn off the gas and raise the thermometer out of the heater.

Fourth. To determine the stem exposure correction.—If a portion of the stem of a mercurial thermometer is at a different temperature from that of the bulb, a stem exposure correction (p. 23) has been shown to be necessary. A similar correction is also necessary in the case of an air thermometer, due to the fact that the air in the stem and in C is not at the same temperature as that in the bulb when the latter is placed in ice, steam, etc. The amount of this correction depends upon the ratio of the volume of air contained in C and the stem, to the volume of the bulb. A comparatively rough determination of this ratio suffices to compute the correction with sufficient accuracy. The volume of the exposed stem is usually made as small as possible, so that the resulting correction shall be small. The volume V of the bulb is to be computed from three measurements of its circumference (or diameter), taken in planes at right angles to each other. The circumference may be measured with a tape, or better by a strip of paper, the length of which is afterwards measured on a scale. In work of extreme accuracy the volume may be determined by filling the bulb with water or mercury and weighing. The volume of the capillary which extends into the heater may be neglected in comparison with the volume of the bulb.

To determine the volume v of the air in C above the mercury, and of the capillary *outside* of the heater, raise D above C and *carefully* open S into position 3, until the mercury completely fills C and the capillary, up to the point where it enters the heater. Close S as in 5, figure 6. Finally turn the cock into position 4 and allow the mercury to run out into a previously weighed small beaker until the meniscus just reaches the reference point. From the weight of this mercury the volume v may be at once computed. Weighings to one per cent. are sufficiently accurate.

Fifth. On completing the experiment be sure that the apparatus is left as follows: C should be raised so that the bulb is out of the heater. The mercury in C should stand two or three centimeters below the reference point, to prevent its soiling the inside of the tube where the readings are taken. The stop-cock S should be closed, and D raised above C, so that there may be no tendency for air to leak into the apparatus at the cock S.

Computation.—The temperature of the steam is to be computed from the data obtained.

By the combined laws of Boyle and Gay-Lussac we have for a perfect gas, and practically also for hydrogen, oxygen, nitrogen, and air,

$$p_t v_t = p_o v_o (1 + at) \qquad (1)$$

where a is the coefficient of expansion of the gas and t is its temperature expressed in degrees centigrade. If $v_o = v_t$, we have, solving the equation for t,

$$t = \frac{p_t - p_o}{a\, p_o} \qquad (2)$$

from which t may be computed at once when the pressure p_o at zero degrees, and the pressure p_t at $t°$, together with the coefficient of tension a (equal practically to the coefficient of expansion) of the gas is known. For dry air $a = 0.003670$.

This simple formula does not apply exactly, however, to the data taken in the preceding experiment, for the condition $v_o = v_t$ is not exactly fulfilled; *i.e.*, the total mass of gas is not brought to the same volume at 0° and at $t°$, for the following reasons:—

First, the volume of the bulb increases when heated from 0° to $t°$, owing to the expansion of the glass; and second, the exposed portion of the air in C and in the capillary does not occupy the volume it would if it were brought to 0° and $t°$, respectively. It is necessary, therefore, to obtain correct expressions for v_o and v_t and to substitute them in the general formula (1) before solving for t.

Let H_1, H_2 = the reduced barometer readings at the time of ice and steam readings, respectively;

h_1, h_2 = the mean manometer pressures reduced to 0°, computed from the observations taken at the temperature of ice and steam, respectively;

t_1, t_2 = the temperature of the mercury columns at the time of taking the ice and steam readings, respectively;

V = the computed volume of the bulb, which may be assumed with sufficient approximation to be its volume at 0°;

v = the volume of exposed stem;

k = the mean coefficient of cubical expansion of glass, = 0.000027 per degree centigrade.

The height h_1 of the mercury column $D_1 - C_1$ reduced to 0° is

$$h_1 = \frac{D_1 - C_1}{1 + 0.000181 t_1}$$

$$= (D_1 - C_1)\ (1 - 0.000181 t_1) \text{ approximately.}$$

Note that this may be negative, which means that the pressure of the gas at 0° is less than one atmosphere.

Similarly

$$h_2 = (D_2 - C_2)\ (1 - 0.000181\ t_2) \text{ approximately.}$$

As the quantities D and C are mean results of observations taken to one-tenth of a millimeter they should be carried *in the* *computation* to hundredths of a millimeter. It will be seen that the reduction to 0° affects only the last two places of significant figures in h, hence three significant figures in the computation of the correction term are sufficient.

The final reduced pressures p_o and p_t are

$$p_o = H_1 + h_1$$
$$p_t = H_2 + h_2.$$

We have next to determine the true volume v_o and v_t which the air would have occupied if it had all been at 0° and $t°$, respectively. The air in the exposed stem of volume v at $t_1°$ and pressure p_o would have occupied the volume $\frac{v}{1 + at_1}$ at 0° and pressure p_o. Hence the total volume which the air in the thermometer would have occupied at 0° is

$$v_o = V + \frac{v}{1 + at_1}.$$

Similarly, a volume of air v in the exposed stem at $t_2°$ and pressure p_t would, at $t°$ and under the same pressure p_t, have occupied the volume $\frac{v}{1 + at_2}(1 + at)$.

If the volume of the bulb is taken as V at 0°, at $t°$ it will be $V(1 + kt)$. Hence the volume which the total mass of air would have occupied at $t°$ and pressure p_t is

$$v_t = V(1 + kt) + v\,\frac{1 + at}{1 + at_2}.$$

Substituting these values in equation (1) we have

$$p_t\left[V(1 + kt) + v\frac{1 + at}{1 + at_2}\right] = p_o\left(V + \frac{v}{1 + at_1}\right)(1 + at).$$

Dividing by $V(1 + at)$ and putting $r = \frac{v}{V}$ for brevity this equation becomes

$$p_t\left(\frac{1 + kt}{1 + at} + r\,\frac{1}{1 + at_2}\right) = p_o\left(1 + r\,\frac{1}{1 + at_1}\right).$$

Solving for t

$$t = \frac{p_t - p_o\left(1 + \frac{r}{1 + at_1}\right) + p_t \frac{r}{1 + at_2}}{ap_o\left(1 + \frac{r}{1 + at_1}\right) - ap_t \frac{r}{1 + at_2} - kp_t}$$

$$= \frac{(p_t - p_o)\left[1 + \frac{r}{p_t - p_o}\left\{\frac{p_t}{1 + at_2} - \frac{p_o}{1 + at_1}\right\}\right]}{(ap_o - kp_t)\left[1 - \frac{ar}{ap_o - kp_t}\left\{\frac{p}{1 + at_2} - \frac{p_o}{1 + at_1}\right\}\right]}. \tag{3}$$

If t_1 does not differ much from t_2, so that without introducing an error greater than the experimental error they may be assumed equal, this formula reduces to

$$t = \frac{p_t - p_o}{ap_o - kp_t}\left[\frac{1 + \frac{r}{1 + at_1}}{1 - \frac{r}{1 + at_1} \cdot \frac{a}{ap_o - kp_t} \cdot (p_t - p_o)}\right]. \tag{4}$$

The form of the correction factor in brackets can be further simplified by writing it in the approximate form

$$1 + \frac{r}{1 + at_1} + \frac{r}{1 + at_1} \cdot \frac{a}{ap_o - kp_t} \cdot (p_t - p_o).$$

If, in this correction term, kp_t can be neglected in comparison with ap_o, the correction further simplifies to

$$1 + \frac{r}{1 + at_1} \cdot \frac{p_t}{p_o},$$

and equation (4) becomes

$$t = \frac{p_t - p_o}{ap_o - kp_t}\left(1 + \frac{r}{1 + at_1} \cdot \frac{p_t}{p_o}\right). \tag{5}$$

Notice that formulæ (4) and (5) reduce at once to (2) if the stem exposure correction involving $\frac{v}{V} = r$ and the correction for the expansion of the bulb, kp_t, are negligible.

Coefficient of Expansion of a Gas.—If instead of solving for t, this is known, the above data may be used for determining a, the coefficient of expansion of the gas. Inspection of equation (3) will show that the equation contains a in three places in such a way that a general solution is exceedingly laborious. Since a enters only in small corrections factors in the terms $\frac{1}{1+at_1}$ and $\frac{1}{1+at_2}$, we may assume an approximate value for it in these terms and solve for a in the term at. If the value thus obtained differs too widely from the assumed value of a, a second or even third approximation must be made. One approximation usually suffices. Solving for a in (3) under this assumption we obtain

$$a = \frac{p_t - p_o + kp_t t - p_o \frac{r}{1+at_1} + p_t \frac{r}{1+at_2}}{t\left(p_o + p_o \frac{r}{1+at_1} - p_t \frac{r}{1+at_2}\right)}.$$

If t_1 and t_2 are so nearly the same that they may be assumed equal, this equation becomes

$$a = \frac{(p_t - p_o)\left[1 + \frac{r}{1+at_1} + \frac{kp_t t}{p_t - p_o}\right]}{tp_o\left[1 - \frac{r}{1+at_1} \cdot \frac{1}{p_o} \cdot (p_t - p_o)\right]},$$

or simplifying the correction factor by an approximation,

$$a = \frac{p_t - p_o}{tp_o}\left[1 + \frac{r}{1+at_1}\left(1 + \frac{p_t - p_o}{p_o}\right)\right] + k\frac{p_t}{p_o}.$$

Problems.— 1. How precise should the ratio $\frac{v}{V} = r$ be determined in this experiment in order that the resulting deviation in t (formula 5) shall not exceed 0.3 per cent?

2. What precision in v and V does this correspond to in the actual apparatus used?

3. How precise should a (formula 5) be known for a precision of 0.3 per cent in t, assuming other sources of error negliglible?

PRESSURE AND BOILING POINT, AND PRESSURE OF SATURATED VAPOR

Object.— This experiment is designed to study the change of the boiling point of a liquid under varying pressure, and to illustrate the dynamical or boiling-point method of determining the pressure of saturated vapor at different temperatures. The experiment also furnishes data for illustrating the application of the fundamental thermodynamic formula for the liquid-vapor state to the calculation of the specific volume of saturated vapor.

Discussion.— A pure liquid can exist in equilibrium with its vapor at any definite temperature and one corresponding pressure, or conversely at any definite pressure and one corresponding temperature. If at any temperature the external pressure on a liquid is greater than the pressure of its vapor and the temperature of the liquid be gradually raised, the pressure of the vapor will increase until it is exactly equal to the external pressure when the liquid will begin to boil. The pressure at which a liquid boils is therefore a direct measure of the pressure of its saturated vapor at its boiling point, and the determination of the boiling point of a liquid under different pressures affords an excellent method for the investigation of the pressure of saturated vapor at various temperatures. This is known as the *Dynamical Method* of vapor pressure measurement as distinct from the *Statical Method,* in which the liquid is introduced into the top of a barometer maintained at a known temperature and the depression of the mercury column observed. The Dynamical Method is by far the more accurate, as it is but little affected by slight impurities in the liquid. The Statical Method, on the other hand, is liable to grave errors from this source, particularly if the impurities are very volatile.

The only rigid relation holding between the pressure p and the boiling point t of a liquid, or in other words, between the temperature t and the pressure p of its saturated vapor, is the thermodynamical equation[1]

$$\frac{dp}{dt} = \frac{r}{T\,(v_2 - v_1)}, \tag{1}$$

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[1] For deduction of this formula see any work on Thermodynamics, as Clausius' "Mechanical Theory of Heat," p. 129; Peabody's "Thermodynamics," p. 95; Noyes' "Introduction to Physical Science," p. 161.

where r is the latent heat of vaporization of the liquid at t° C., T the absolute temperature corresponding to t° C., and v_1 and v_2, the corresponding specific volume of the liquid and its saturated vapor, respectively. As these last quantities as well as r are functions of the temperature, the above equation cannot be integrated without making simplifying assumptions, or by the aid of empirical relations between the quantities r, v_1, v_2 and the temperature t. Thus for water, Clausius deduces from Regnault's measurements the following empirical formula for r:—

$$r = 607 - 0.708t.$$

The exact way in which v_2 varies with t, *i.e.*, the form of function $v_2 = f(t)$ is known for only a few vapors. Saturated vapors are often *assumed* to follow Boyle's law, *i.e.*, $v_2 = \frac{RT}{p}$, but this assumption is in most cases only a rough approximation and is liable to lead to erroneous results.

Equation (1) is of great value in computing the specific volume v_2 of saturated vapor when $\frac{dp}{dt}$ and r are known, as v_2 itself can be experimentally determined with accuracy only with great difficulty. The specific volume v_1 of the liquid is approximately equal to unity, and therefore is usually negligible compared with v_2 in any computation. The experimental determination of $\frac{dp}{dt}$ affords therefore a valuable means of finding indirectly the specific volume of saturated vapor.

Equation (1) is not limited to changes from the liquid to the vapor state, but holds equally true for all changes of state, for example the melting of ice to water, in which case r denotes the latent heat of fusion, T the absolute melting point, v_1 and v_2 the specific volumes of the substance in the initial and final states respectively.

Besides the thermodynamic formula (1) between t and p, numerous empirical relations have been proposed by different investigators.[1] Only one will be mentioned here, namely, that proposed by Regnault to represent his classical data on a large number of saturated vapors. By plotting temperatures and corresponding pressures on a large scale, he found the resulting curves could be represented by interpolation formulæ of the type

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[1] Winkelmann, "Handbuch der Physik," vol. ii. 2, p. 702. Ostwald, "Lehrbuch der Allgemeinen Chemie," vol. i. p. 313.

$$\log p = a + b\alpha^{t}$$

where a, b, and α are constants depending on the nature of the liquid, $t = t_c \pm C$, where t_c is the temperature in centigrade degrees and C is a constant. He derives the following values for these constants for the following typical liquids:—

	a	b	$\log \alpha$	C
Water	5.42332	—5.46428	0.9972311—1	+20°
Alcohol	5.54320	—5.01945	0.9972021—1	+20°
Benzene	4.67667	—4.07461	0.9965676—1	+24°

Apparatus.—The arrangement of the apparatus is as follows: An open manometer A is connected through a large air reservoir B, and a return condenser C, with the boiling apparatus D. This consists of a horizontal brass cylinder in the axis of which is sealed a small brass tube closed at one end for containing the thermometer for registering the temperature of the boiler.

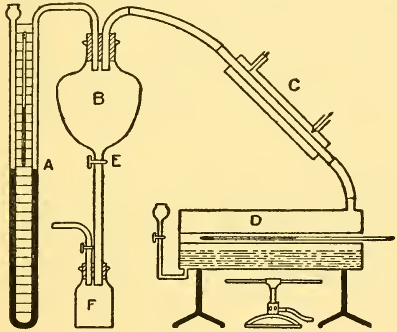

Fig 8.

The pressure in the apparatus may be varied by connecting the reservoir B with a suction or a pressure pump through the bottle F. This is inserted as a trap to prevent water from the water pump from sucking back into the apparatus, and to collect, when the cock E is opened, any water which may have condensed in B.

The boiler is filled with water through a thistle tube gauge attached at one end; the height of water in the boiler is indicated by the position of the water in this gauge which is of glass. The burner is constructed so as to distribute a series of small flames along the length of the boiler.

Procedure. — The following directions apply particularly to water, which is the liquid used in this experiment:—

First. Connect *B* to the suction-pump through the safety bottle *F* and exhaust until the pressure within the apparatus is reduced to about 50–60 mm. Close *E* and the second stop-cock between *F* and the pump. *Then, and not before,* turn off the water. See that the apparatus is air-tight (indicated by a constant reading of the manometer), before proceeding further. If it is not, speak to an instructor.

Second. See that the waste-cock from the condenser is open *before* turning on the water through *C*. It should never be closed. Turn on the water, so that a good stream flows through the condenser.

Third. Bring the water in the boiler to brisk boiling. Only a small flame will be necessary. The boiling point will probably be not more than 15° or 20° above the temperature of the room. As soon as the thermometer indicates a constant temperature, record the temperature and the height of the mercury in the manometer.

Record also the reading of the thermometer attached to the manometer, the height of the barometer, and its temperature. Usually two barometer readings, one at the beginning of the experiment and one at the end, will be sufficient, unless it is changing very rapidly, when readings should be taken directly before or after recording the manometer. This completes all data necessary for the first set of observations.

Next allow air to enter the apparatus through *E* until the manometer indicates an increase of pressure of about 100–150 mm. By thus increasing the pressure, boiling will instantly cease until the temperature has risen to the new boiling point, when a second series of observations similar to the above is to be taken. Determine in this manner the boiling points corresponding to six or seven different pressures, taking the last observation at atmospheric pressure, or a little above it. This last may be done by forcing air into the apparatus at E by means of a small force pump.

Computation.— Correct the observed boiling points for thermometric errors (see table of corrections for thermometer). Reduce the corresponding pressures in millimeters of mercury to 0° C., assuming the manometer scale (of seasoned boxwood) to be correct at ordinary temperatures. Plot the reduced data with pressures as ordinates, and temperatures as abscissæ. This will show graphically the relation between the pressure and boiling point of the liquid, or, in other words, the variation of the pressure of its saturated vapor with the temperature.

Calculate by equation (1) the specific volume of saturated steam, v_2, at 100° C., and compare it with the value given in Peabody's Steam Tables or in Table VIII., Appendix.

Note.— To obtain the value of $\frac{dp}{dt}$ at 100° where the vapor pressure curve will be seen to be nearly linear over a considerable range of temperature, compute the approximate value $\frac{\Delta p}{\Delta t}$ by taking two points on the actual curve near 100°, instead of attempting to draw a tangent to the curve at that point. With a thermometer reading to 0.001° C., and a sensitive manometer the value of $\frac{\Delta p}{\Delta t}$ can be obtained directly from the observed data on two boiling points with considerable accuracy.

In solving for v_2 *in equation* (1) *note that* dp *and* r *must be expressed in corresponding units,* e.g., *dynes per sq. cm. and ergs respectively.*

Problems.— 1. At what pressure will an error of 0.1 mm. in p begin to affect the corresponding value of t by 0.1° C.? By 0.05° C.? (Solve by inspection of Steam Tables.)

2. Compute from formula (1) the melting point of ice under a pressure of ten atmospheres.

Given: latent heat of fusion of ice = 80 calories,
specific volume of ice = 1.09 c.cm.,
specific volume of water at 0° C. = 1.00 c.cm.

CALORIMETRY

GENERAL DISCUSSION

This discussion should be carefully studied before beginning any experimental work on calorimetry.

Calorimetry is the process of measuring quantities of heat. The process in general consists in transforming a quantity of some form of energy, as electrical energy or mechanical energy into heat energy, in a vessel called the calorimeter which contains water, or some other appropriate liquid, and in measuring the resulting phenomenon produced. By the principle of the conservation of energy the quantity of energy thus introduced is equal to the sum of all the energy changes resulting in the calorimeter, supposing the latter to receive no other energy from, or give no energy to, the surroundings. The various calorimetric methods devised seek to reduce these energy changes to the simplest and most accurately measurable forms.

The most common method, known as the Method of Mixtures, consists in introducing the energy directly into a calorimeter containing a known weight of water, in which case the phenomenon to be measured is simply the resulting rise of temperature of the calorimeter and water. If this be accurately determined and the weight and specific heat of the water, together with the weight and the specific heat of the calorimeter and all its parts be known, the quantity of heat energy introduced into the calorimeter is given at once by the expression:—

$$q = [w_o s_o + \Sigma w_k s_k] \Delta t \qquad (1)$$

where w_o and s_o is the weight and specific heat, respectively, of the water (s_o may usually be taken equal to unity), w_k, s_k are the weights and specific heats, respectively, of the materials constituting the various parts of the calorimeter, and Δt is the rise of temperature common to them all.

Simple as the method is in theory the quantities involved in the measurements and the conditions to be observed are such that calorimetric measurements, which are reliable to 0.1 per cent, are among the more difficult laboratory processes.

Units.—The unit of heat energy is the *calorie,* or more specifically the *gram-calorie* or *kilogram-calorie.* It is defined as the quantity of heat required to raise a unit mass (one gram or one kilogram) of water through one degree centigrade. It has been experimentally found that this quantity varies with the temperature, hence, in order to definitely define the calorie, it is necessary to further state the particular temperature to which it is referred. Owing to the choice of various temperatures by different observers, several calories have come into use with the inevitable result of introducing great confusion in the comparison of the results of different investigators.

The universal adoption of one definite unit is much to be desired; but until the use of such a unit becomes general, a definite statement of the calorie used should always be made in expressing experimental results which have any claim to accuracy. The following different calories should not be confused:

1. Zero Degree Calorie:—The quantity of heat required to raise one gram of water from 0° to 1° C.
2. Ordinary Calorie: cal.—The quantity of heat required to raise one gram of water from 15° to 16° C., this being taken as the mean room temperature.
3. Mean Calorie or Ice Calorimeter Calorie:—The one-hundredth part of the heat required to raise one gram of water from 0° to 100° C.
4. Ostwald's Calorie: K.—The quantity of heat required to raise one gram of water from 0° to 100° C.
5. Berthelot's Large Calorie: Cal.—1000 times the ordinary calorie as defined above.

The zero degree calorie is of little or no practical use, and is to be regarded only as a classical definition introduced by Regnault. The specific heat of water in the neighborhood of zero degrees is very difficult to determine with accuracy, and hence were this unit adopted, its value would be more uncertain than the calorimetric measurements themselves.

The ordinary calorie denoted by cal. is that most frequently adopted, as most calorimetric measurements are carried out at "room temperature," which is variously taken from 15° to 18° C. The calorie is nearly constant throughout this range of temperature.

The mean calorie or ice calorimeter calorie has been used chiefly in expressing results obtained with the ice calorimeter, which is usually calibrated by the quantity of heat given out by water cooling from 100° to 0°. This unit does not differ from the ordinary calorie by more than 0.14 per cent.

Ostwald, following the suggestion of Schüller and Wartha, has proposed the 100 times greater unit denoted by K, as being not only of a more convenient magnitude for expressing thermochemical data, but also more rational than the somewhat arbitrary ordinary calorie. Its advantage as a thermo-chemical unit of heat measurement will be seen from the following example: The heat of combustion of one atomic weight (12 grams) of carbon to carbon dioxide is 97000. cal., but as thermochemical data are seldom reliable to more than 0.1 per cent, *i.e.*, to four significant figures, the superfluous ciphers necessary to fix the decimal point may be avoided by using the larger Ostwald calorie K, and writing the above quantity of heat 970 K. For the same reason Berthelot adopts the still larger unit 1 Cal. = 1000 cal. for expressing the results of his thermochemical investigations. Ostwald's calorie is in general use in Germany; Berthelot's in France.

From the most recent determinations of the variation of the specific heat of water with the temperature, the following relations hold between the various calories above defined:

1 zero degree calorie = 1.008 ordinary (15° – 16°) calories.[1]
1 mean calorie = 1.0014 ordinary (20°) calories.[2]
In absolute units 1 ordinary (15°) calorie = g J ergs
= 980.6 × 42730 ergs
= 4.190×10^7 ergs

J = 42730 gram-centimeters at sea level, 45° latitude (g = 980.6), and referred to the ordinary calorie.

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[1] Lüdin, Wied. Beiblätter, **20**, 764, 1896.
[2] Callander and Barnes, Phys. Rev., **10**, 213, 1900.

Sources of Error.— The principal sources of error which arise in calorimetry, and which in accurate work must be corrected for or rendered negligible by special adjustment of the conditions of the experiment, are the following:

First.— Errors in Thermometry. The rise of temperature in a calorimetric measurement is in most cases small — a few degrees only—hence thermometers sensitive to at least 0.01° C. are usually required. When the rise of temperature is very small, a few tenths of a degree only, thermometers indicating differences of temperature of 0.005° or even 0.001° are required. The calibration errors and scale unit of such *calorimeter thermometers* must be carefully determined, as well as the corrections for all other thermometers used in the work.

Second.—Errors arising in the determination of the heat capacity or water equivalent of the calorimeter. By this is understood the value of the term $\Sigma w_k s_k$ in formula (1), that is, the total heat capacity of all parts of the calorimeter and its contents, exclusive of the water. This is called the "water equivalent" of the calorimeter, since this number of grams of water (specific heat unity), would be raised through the same temperature as the material in question by the same quantity of heat. If the weights and specific heats of all parts of the calorimeter are known with sufficient accuracy, the water equivalent can be computed at once, in which case the calorimetric measurement is a primary one. If, on the other hand, these data are not known with sufficient accuracy, the water equivalent has to be experimentally determined, and the measurement is then a secondary one.

It is always desirable that the water equivalent of the calorimeter be as small as possible in comparison with the weight of water used. The material of which a calorimeter is constructed should therefore be as thin as is consistent with rigidity, and should have as low a specific heat as possible. It is also desirable that it have a high thermal conductivity in order to assume quickly the mean temperature of the water, and for reasons mentioned below, it should also be capable of a high polish. The metals best fulfilling these conditions will be seen from the following table:

	Sp. Heat	*Density*	*Product*	*Thermal Conductivity*
Platinum	0.032	21.5	0.69	. . .
Silver	0.056	10.5	0.59	109.
Nickel	0.11	8.9	0.98	. . .
Copper	0.093	8.9	0.83	82.
Glass	0.20	2.4	0.48	0.13
Mercury	0.033	13.6	0.45	2.

It appears from the above table that silver and platinum are the metals best adapted for constructing calorimeters, both on account of their low specific heat and high thermal conductivity. Platinum is the more durable, and is especially adapted for thermo-chemical experiments involving the use of acids and bases which attack other metals. Gold-plated silver calorimeters may, however, often be used for such experiments and are much cheaper. Glass is not to be recommended on account of its very low thermal conductivity, which is only about one-thousandth that of silver. Nickel, or nickel-plated copper may be used with non-corrosive liquids.

The exact computation of the heat capacity of the bulb and immersed portion of the calorimeter thermometer is usually impossible, as the weight of mercury and glass cannot separately be known. It happens fortunately, however, that the heat capacity of an equal volume of mercury and of glass is nearly the same, 0.45 and 0.48, respectively (see above table). Hence the heat capacity of the immersed portion of a thermometer may be computed with very close approximation as the product of the mean of these values, namely 0.465, times the total volume of the part of the thermometer which is immersed.

Third.—Errors arising from exchange of heat by conduction and radiation. As every calorimetric measurement requires time for its completion, the calorimeter and contents will gain or lose a certain quantity of heat by conduction and radiation from or to the surroundings, unless the surroundings are maintained exactly at the temperature of the calorimeter throughout the experiment. As the temperature is continually changing, this condition cannot in general be realized, and hence corrections must be made for these two sources of error. Gain or loss of heat by conduction may be practically eliminated by supporting the calorimeter on three non-conducting points such as corks, or resting it on a net-work of strings. If the calorimeter could be vacuum jacketed, practically all exchange of heat by conduction or convection would be eliminated.

The errors due to radiation are however more serious, and constitute one of the most troublesome sources of error met with in calorimetric work. The quantity of heat radiated (or absorbed) by a surface is proportional to:

1. The area of the radiating surface.
2. A constant depending on the nature of the radiating surface.
3. The difference of temperature between the surface and surroundings.
4. The time during which radiation takes place.

The radiating surface should therefore be as small as is consistent with the required volume. Calorimeters are usually cylindrical in form, and hence for a closed calorimeter, *i.e.*, one provided with a cover, the best relative dimensions would be height = diameter. They are usually, however, made with the diameter considerably smaller in proportion than this, in order to diminish the area of the surface of the liquid exposed to evaporation. Calorimeters of less than 400–500 cubic centimeters capacity are not to be recommended except for special work.

The numerical value of the radiation factor depends upon the degree of polish of the radiating surface. If the surface is rough and tarnished the coefficient is large, being a maximum for a "perfectly black" surface. On the other hand, it may be reduced to a very small value by giving the surface of the calorimeter the highest possible burnish. To still further reduce this source of error the calorimeter should be enclosed within a second vessel or jacket whose inner walls are also burnished. The third and fourth factors show that the temperature of the calorimeter should not differ very greatly from that of the surroundings, and that the duration of the experiment should be as short as is consistent with other conditions. If the difference in temperature between calorimeter and surroundings amounts to more than about 10° C., the conditions enumerated above, which are a statement of Newton's Law of Cooling, no longer hold true. In what follows it will be assumed that the temperature difference does not exceed this amount.

Cooling Correction. — Let us now investigate how the heat lost or gained by radiation may be allowed for or eliminated. This correction is generally known as the "cooling correction." A calorimetric measurement consists in general of the three following continuous operations:

First. Preliminary readings of the temperature of the water in the calorimeter every half minute for at least five minutes immediately preceding the operation proper.

Second. The operation proper, consisting of introducing heat into the calorimeter, *e.g.*, dropping in a hot substance as in "Specific Heat," or passing in steam as in "Latent Heat," or heating a coil of wire by an electric current as in "Mechanical Equivalent."

Third. Final temperature readings after the operation has ended, continuing at half minute intervals for at least another five minutes (better for eight or ten minutes), these data to serve for determining the final rate of gain or loss of heat.

The preliminary readings furnish data for finding the temperature (which in general cannot be read), at the moment of beginning the operation, and for determining the rate of gain or loss of heat of the calorimeter at that time.

If the data thus obtained be plotted with temperatures as ordinates and times as abscissæ, curves of the following general type will be obtained:

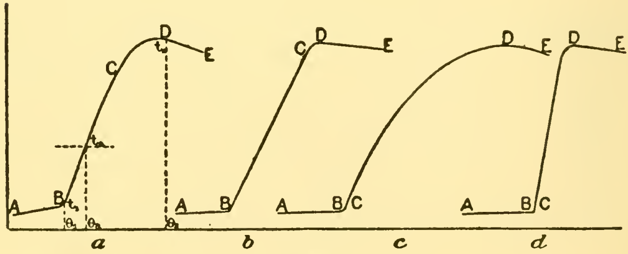

Fig. 9.

Curve *a*, figure 9, represents the general case in which the calorimeter gains heat from the surroundings from *A* to *B;* the operation begins at *B* and heat is developed within the calorimeter until the temperature has risen to *C;* at *C* the operation ceases, but if certain parts of the calorimeter or its contents have become heated above the mean temperature of the calorimeter, there will then result a further rise of temperature due to the equalization of temperature between such hot parts and the water of the calorimeter, or between the water and the metal of the calorimeter itself. This will be represented by the portion of the curve *CD, D* being the temperature at which the whole calorimeter and contents arrive at a uniform temperature. If *D* is above the temperature of the surroundings, as is usually the case, the calorimeter will then give up heat to the surroundings by radiation at a uniform rate represented by the straight line *DE*. The temperature *D* marks the beginning of this straight line which should be tangent to the curve *CD* at *D*.

In certain operations, as in the development of heat in the calorimeter by heating a coil of wire by a current (as in "Mechanical Equivalent"), or in condensing steam in the calorimeter (as in "Latent Heat") the portion *CD* of curve *a* may become very small as in curve *b*, very little rise of temperature occurring after the operation ceases. On the other hand, when a heated mass is suddenly introduced into the calorimeter (as in "Specific Heat"), the whole operation consists of an equalization of temperature, and *B* and *C* fall together. In this case *CD* may take any form between a curve of type *c* and a nearly straight line, as in curve *d*, according to the velocity with which the equalization of temperature takes place. If the substance introduced is a good conductor and has a large surface compared with its mass, *e.g.*, if it is cut up into small pieces, the curve will be of the form *d*. If, however, the substance is a single large mass of poorly conducting material, such as vulcanite or glass, the curve will take the form *c*.

In order that curves similar to the above types may be obtained, it is absolutely essential that the stirring be so efficient and the thermometer so placed in the calorimeter, that the temperature indicated is the mean temperature of the calorimeter and contents as a whole, at the time of observation. Irregularities in the curves can almost invariably be traced to failure to fulfil one or both of these conditions. Thus a hump in the curve at D indicates that the bulb of the thermometer has been unduly heated by too close proximity to the source of heat. In all such cases the data should be rejected.

Demonstration. — It will now be shown how the cooling correction can be computed, first for the general case represented by a, figure 9, and then for several special cases.

General case. Let r_1 be the rate of exchange of heat of the calorimeter in degrees per minute, reckoned plus for gain and minus for loss of heat, as deduced from the tangent of the straight line best representing the temperatures between A and B. Let $t_1 =$ the temperature at the instant the operation begins at B, and θ_1, the corresponding time. Note that since one observer can seldom read this temperature at the time of performing the operation, it must be obtained by extrapolating the line AB to the time θ_1, at which the operation begins. Similarly let r_2 be the rate of exchange at the final temperature t_2 at the time θ_2, *i.e.*, the temperature and time corresponding to the point D. Let t_a be the mean temperature of the surroundings. The calorimeter then gains heat from the surroundings up to a time θ_a when its temperature reaches that of the surroundings, after which it begins to lose heat.

By Newton's law of cooling the rate at which a body gains or loses heat by radiation is directly proportional to the difference of temperature between the body and its surroundings. Applied to a calorimeter, if its heat capacity does not sensibly vary during the operation, we may assume that its rate of loss or gain of *heat* is proportional to its rate of rise or fall of *temperature*. This assumption is made in the following demonstration:

Let the temperature change per unit time at any temperature t be denoted by r. Then $r = a\,(t_a - t)$ where a is a constant depending on the nature of the given calorimeter.

The total rise in temperature due to radiation during the interval from θ_1 to θ_2 will be therefore

$$\int_{\theta_1}^{\theta_2} r d\theta = a \int_{\theta_1}^{\theta_2} (t_a - t)\, d\theta = a \left[t_a \int_{\theta_1}^{\theta_2} d\theta - \int_{\theta_1}^{\theta_2} t d\theta \right]$$

$$= a \left[t_a (\theta_2 - \theta_1) - \int_{\theta_1}^{\theta_2} f(\theta)\, d\theta \right]. \qquad (1)$$

But $\int_{\theta_1}^{\theta_2} f(\theta)\, d\theta$ is the area bounded between the curve, the ordinates at θ_1 and θ_2, and the axis of θ. Its approximate value may be found by computing the mean value of the ordinate t_m, *i.e.*, the mean value of the temperature between t_1 and t_2, and multiplying by the time interval $\theta_2 - \theta_1$.

$$t_m = \frac{1}{n-1} \left[\frac{a}{2} + b + c + \ldots + \frac{n}{2} \right]$$

$$= \frac{1}{n-1} \left[\frac{1}{2} t_1 + b + c + \ldots + \frac{1}{2} t_2 \right]$$

where a, b, … n are temperature readings taken from the plot or data at the time θ_1 and each following half minute up to θ_2, respectively. Hence the approximate value of formula (1) becomes

$$\int_{\theta_1}^{\theta_2} r d\theta = a \left[t_a (\theta_2 - \theta_1) - t_m (\theta_2 - \theta_1) \right] = a (t_a - t_m)(\theta_2 - \theta_1),$$

which taken with opposite sign is the correction due to radiation to be added to the observed rise in temperature. The corrected rise of temperature is therefore

$$t_2 - t_1 + a (t_m - t_a)(\theta_2 - \theta_1). \qquad (2)$$

This formula is general and can be applied to a plot of calorimetric data of any form.

The radiation factor a can be determined at once from the simultaneous equations

$$r_1 = a (t_a - t_1)$$
$$r_2 = a (t_a - t_2)$$

whence

$$a = \frac{r_1 - r_2}{t_2 - t_1}.$$

The mean temperature of the surroundings t_a can be observed, or it may be computed from the above equations as

$$t_a = t_1 + \frac{r_1}{a} \text{ or } t_a = t_2 + \frac{r_2}{a}$$

It is evident from the above deduction that the correction term in (2) becomes zero when $t_m = t_a$. By preliminary experiments on the actual rise of temperature occurring in the experiment, it is possible to calculate what the initial temperature of the water below the temperature of the surroundings should be in order that this condition may be fulfilled, and the extra labor thus involved is in many experiments warranted.

Special Case I.—The Rumford Method. The total rise of temperature is at a uniform rate.

If the rise of temperature is uniform during the whole operation, and the part *CD* of the curve is negligible compared with *BC*, *i.e.*, *C* and *D* practically coïncide, the mean temperature t_m will evidently be $t_m = \frac{t_1 + t_2}{2}$. Hence if the initial temperature of the water is brought to just as many degrees *below* the temperature of the surroundings t_a, as the final temperature is allowed to rise *above* this temperature, then $t_m = t_a$ and the gain and loss of heat during the operation will practically offset each other. The observed rise of temperature will therefore be the correct one. This is known as the *Rumford Method* of eliminating the cooling correction. It is to be especially noted that it is only applicable when the *entire* rise of temperature is at a uniform rate, for only then is

$$t_m = \frac{t_1 + t_2}{2} = t_a\,.$$

If in the above case the initial and final temperatures are not so adjusted with respect to the surroundings that $t_m = t_a$, *i.e.*, r_1 is not exactly equal to r_2, it is easy to show that the correction to be applied (*i.e.*, added) is

$$-\frac{r_1 + r_2}{2}(\theta_2 - \theta_1). \tag{3}$$

For suppose the data to be represented as in figure 10. If t_a is the mean temperature of the surroundings, the loss of heat during the time the temperature rises from t_a to t_2 will be exactly compensated by gain of heat during the interval it is rising from some temperature t_x to t_a where t_x is just as far below t_a as t_2 is above it. The resultant correction will therefore be the gain of heat during the period θ_1 to θ_x while the temperature is rising from t_1 to t_x. This may be found at once by substituting in the general formula (1). We obtain for the correction

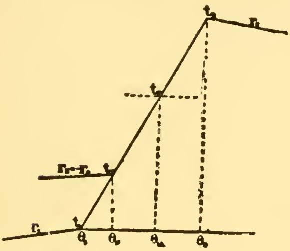

Fig. 10.

$$\frac{r_1 - r_x}{t_x - t_1}(t_m - t_a)\ (\theta_x - \theta_1).$$

This may be written

$$(r_1 - r_x)\left(\frac{t_1 + t_x}{2} - t_a\right)\left(\frac{\theta_2 - \theta_1}{t_2 - t_1}\right),$$

$$\text{as } t_m = \frac{t_1 + t_x}{2} \quad \text{and} \quad \frac{\theta_x - \theta_1}{t_x - t_1} = \frac{\theta_2 - \theta_1}{t_2 - t_1}.$$

$$\text{But} \qquad t_x = t_a - (t_2 - t_a) = 2t_a - t_2;$$

hence the correction reduces at once to

$$-\frac{r_1 - r_x}{2}(\theta_2 - \theta_1) = -\frac{r_1 + r_2}{2}(\theta_2 - \theta_1).$$

r_1 and r_2 are obtained by finding the tangent of the angle which the straight lines best representing the preliminary and final readings, make with the axis of θ respectively. They are to be algebraically added, gain of heat being reckoned positive, and loss of heat negative.

Special Case II.—The rise of temperature from B to C is uniform, but the portion of the curve CD, although small, is not negligible compared with BC, figure 11. Then, since the rate of exchange during the interval CD is practically r_2, the temperature would have risen to C' had the equalization of heat from C to D been instantaneous, where C' is the intercept of DE extended back to the ordinate through C. Call this temperature t'_2. The cooling correction for the interval BC is that already deduced under Special Case I. Hence the corrected rise of temperature will be

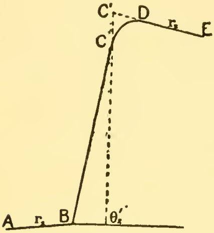

Fig. 11.

$$t'_2 - t_1 - \frac{r_1 + r_2}{2}(\theta'_2 - \theta_1), \tag{4}$$

where θ'_2 is the time corresponding to t'_2, *i.e.*, the abscissa of the point C or C'.

Special Case III.—The rise of temperature during the operation is uniform, and the initial temperature is so adjusted that $r_2 = 0$, *i.e.*, DE is horizontal. This is evidently a special case of the above, and the corrected rise of temperature is

$$t'_2 - t_1 - \frac{r_1}{2}(\theta'_2 - \theta_1). \tag{5}$$

Special Case IV.—When the equalization of temperature CD is slow, as in c, figure 1, or constitutes a considerable portion of the total rise, it is advantageous to so adjust the initial temperature that the final temperature t_2 coincides as nearly as may be with the temperature of the surroundings, *i.e.*, so that $r_2 = 0$. The correction is to be computed by the general formula. This procedure not only reduces the magnitude of the rate r_2 most affecting the correction, but also serves to determine it with greater accuracy.

SPECIFIC HEAT OF SOLIDS

THE METHOD OF MIXTURES

Object.— The object of this experiment is to determine the specific heat of a solid substance by the Method of Mixtures to about 0.5 per cent. The experiment illustrates the general procedure in a simple calorimetric measurement, the further application of various matters discussed under thermometry, and the method of applying the cooling correction.

Apparatus.— The apparatus used is shown in figure 12. It consists of an open calorimeter *C* of burnished nickel-plated copper which rests on cork supports inside a similar nickel plated vessel or jacket; a stirrer *S* of the same material is also provided. The calorimeter thermometer *T* for measuring the rise in temperature is held in place by a support fixed to the jacket. The calorimeter is protected from the heat and the steam of the boiler by a double walled screen *W* open at the bottom and top to permit free circulation of air.

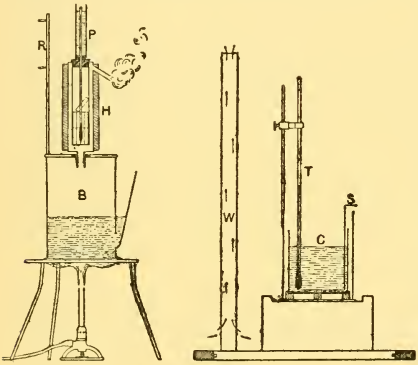

Fig. 12.

The heating apparatus consists of a boiler *B*, on the top of which is placed the heater proper, *H*. This consists of two concentric brass cylinders joined at the top. The inner cylinder is closed at the bottom and forms a receptacle for the substance to be heated. The bottom of the larger outer cylinder terminates in a conical tube by means of which it is connected with the boiler. The steam passes up around the inner cylinder and escapes through a small side tube at the top. The outer cylinder is heavily lagged with felt to prevent loss of heat and to facilitate handling while hot. The substance to be heated is placed in the inner cylinder which is then closed by a cork or rubber stopper, through which the bulb of the thermometer for taking the temperature of the substance should pass easily. The upper part of this thermometer which projects beyond the stopper is enclosed in a "stem exposure" tube P (see page 23) filled with water, the temperature of which is determined by an auxiliary thermometer not shown in the figure. The corrections for all thermometers used with the apparatus are given on their cases.

Procedure.— Place the substance in the heater and begin heating it up *at once*, as the longest part of this experiment consists in allowing the substance to reach a uniform high temperature. See that the graduations on the thermometer which is used for taking the temperature of the substance are distinctly visible from 90°–100°. If not, rub in some lead with a soft pencil. If this precaution is not observed, difficulty may be encountered in reading the thermometer afterwards when it is immersed in water. Arrange the thermometer and stem exposure tube as shown in the figure. Record where the thermometer enters the tube as datum for determining the stem exposure correction.

While the substance is heating, weigh the calorimeter and the stirrer, dry. Next fill the calorimeter to about three-fourths its capacity with water and weigh. Ordinary tap water may be used. As the exact rise of temperature is unknown, it is not possible in a single experiment to adjust the initial temperature of the water to the best point below that of the surroundings. The apparatus is so designed, however, that the rise of temperature with the substances employed is about 4° or 5°. Moreover the substance used is metal, and is divided into a number of small pieces so that the equalization of heat between it and the calorimeter is very rapid. The plotted data will give a curve similar to that in *d*, figure 1. With this knowledge it is best to adjust the water in the calorimeter to about two or three and one-half degrees below the temperature of the surroundings, *i.e.*, the temperature indicated by a thermometer (the calorimeter thermometer may be used), placed within and touching the side of the jacket surrounding the calorimeter. If a preliminary experiment should show the equalization of temperature to take place slowly, as in the case of poorly conducting substances, the initial conditions should be adjusted as described under special case IV, p. 54.

A blank form should next be prepared for recording data with one column for the time in hours, minutes and seconds, and another for the corresponding temperature readings. The times *should be written down beforehand* in half-minute intervals for a period of at least fifteen minutes. Such a previously prepared blank is of great assistance in taking down data of this kind, as it lessens the labor during the operation, distracts the attention of the observer less from his temperature readings, and in case any observations are omitted, greatly lessens the confusion in recording subsequent data in their proper place.

When the temperature of the heated substance becomes constant, and *not before*, begin taking the thermometer readings. Record first with the calorimeter thermometer the temperature t_s of the surroundings (inside of the jacket). Then place the thermometer in the calorimeter and with *regular* and *constant* stirring (a long and not too rapid up and down motion), begin the "preliminary readings," continuing the same for at least five minutes. The thermometer should be read to 0.01° C. Suppose, for illustration, that the first reading was at 2 h. 35 min. 0 sec. At the end of five minutes, *i.e.*, after the reading at 2 h. 40 min. 0 sec., suspend readings of the calorimeter thermometer and prepare for the operation proper. First, read the temperature of the stem exposure thermometer and remove it. Second, read very carefully the temperature of the hot substance. Third, remove the thermometer with its attached stem exposure tube from the heater and place it in the support *R*. In this operation do *not* remove the cork closing the heater; otherwise the substance will become somewhat cooled. These operations can be easily made without undue haste during the minute from 2 h. 40 min. 0 sec. to 2 h. 41 min. 0 sec. A few seconds before the latter minute expires remove the heater from the boiler, and exactly on the 41st minute take out the stopper and pour the hot substance into the calorimeter. The calorimeter thermometer should be so placed that the substance is poured *away* from its bulb, not only to prevent accident, but also to avoid local heating of the water in the neighborhood of the bulb. It is safer during this operation to raise the thermometer out of the calorimeter entirely. *Immediately after the transfer resume stirring the contents of the calorimeter as thoroughly as possible.* The success of the experiment from this point on depends largely on the efficiency of the stirring. Read the calorimeter thermometer one-half minute after the operation, *i.e.*, at 2 h. 41 min. 30 sec., and on every following half-minute for eight or ten minutes, stirring of course constantly. This completes the calorimetric observations. There still remains only to determine the weight of the substance dry.

At the conclusion of the experiment replace the substance (dry), all three thermometers, and stem exposure accessories, in their case and return the same to the instructor.

Computation.— *First.* Make a plot of the temperature and time observations, and from it determine the cooling correction. The form of the curve obtained will determine which method, pp. 52–54, it is best to apply. The temperature t_1 of the calorimeter at the instant of inserting the hot substance is also to be determined from the plot.

Second. Compute the temperature t_s of the hot substance at the time of introduction into the calorimeter, applying calibration and stem exposure corrections to the observed reading. See p. 23.

Third. Calculate the water equivalent of the calorimeter, $k = w_1 s_1 + w_2 s_2 + \dots = \Sigma\, w_k s_k$. The specific heat of the material of the calorimeter may be taken as 0.095. The water equivalent of the immersed thermometer is readily shown to be negligible in this experiment.

Fourth. Compute from the above data the specific heat of the substance. The formula for computing the specific heat by the method of mixtures is

$$s = \frac{(w_o + k)\ (t_2 - t_1)}{w\ (t_s - t_2)},$$

where s = specific heat of substance;
w = weight of substance;
w_o = weight of water;
w_1, w_2, etc. = weights of various parts of calorimeter;
s_1, s_2, etc. = specific heats of various parts of the calorimeter;
$t_s - t_2$ = corrected fall of temperature of substance;
$t_2 - t_1$ = corrected rise of temperature in calorimeter.

Precision Discussion.— If each parenthesis in the above formula be regarded as a single independent variable, the formula for s can be treated as a simple product and quotient of four variables, namely, the rise of temperature of the calorimeter and contents, the fall of temperature of the substance, the weight of the substance, and the weight of the water plus the water equivalent of the calorimeter. Suppose it is desired to determine s to 0.5 per cent. What will be the allowable deviations in these four compoments?

Solving for "Equal Effects" we have at once

$$\frac{\delta(w_o + k)}{w_o + k} = \frac{\delta(t_2 - t_1)}{t_2 - t_1} = \frac{\delta w}{w} = \frac{\delta(t_s - t_2)}{t_s - t_2} = \frac{1}{\sqrt{4}} \cdot \frac{\delta s}{s} = \frac{0.005}{2} = 0.0025.$$

All four factors must therefore be measured to 0.25 per cent. With the apparatus provided the following approximate values may be assumed for illustration. Suppose

$w = 300$ gms. then $\delta w = 0.75$ gm.
$w_o + k = 500$ gms. " $\delta(w_o + k) = 1.2$ gms.
$t_2 - t_1 = 20° - 15° = 5°$ " $\delta(t_2 - t_1) = 0.013°$.
$t_s - t_2 = 100° - 20° = 80°$ " $\delta(t_s - t_2) = 0.20°$.

It is evident from these results that the greatest difficulty in attaining the desired degree of precision is in the determination of the rise of temperature in the calorimeter. For a rise of five degrees, the allowable deviation after all thermometric and radiation corrections have been applied, must not be greater than 0.013° and this cannot be attained without considerable skill and care.

To measure the fall in temperature of the substance to 0.2° through a range of 80° necessitates a careful determination of the correction of the thermometer used, at 100°.

On the other hand the weight of the substance w can be determined with ease to ten times the required precision with an ordinary balance. If therefore w be weighed to 0.07 gram or even 0.1 gram, the error in it may be assumed negligible, and hence the allowable deviations in the temperature measurements may be a little greater without increasing the deviation of the final result to more than 0.5 per cent. The deviation in $w_o + k$ may also be made negligible as the following consideration will show.

$$k = w_1 k_1 + w_2 k_2 + \ldots$$

Suppose, as in the experiment, that the specific heat of all the material composing the calorimeter, stirrer, etc. is 0.095 and that its total weight is about 100 grams. Then $k = 9.5$. If the value 0.095 is reliable to a per cent, *i.e.*, about one unit in the third decimal place, and if the calorimeter stirrer, etc., be weighed to one per cent or better 0.5 per cent, *i.e.*, the nearest half gram, the value of $k = 9.5$ will be reliable to one unit in the tenths place. Hence if the water be weighed to the nearest 0.1 gram or 0.2 gram as may easily be done, the resulting deviation in $w_o + k =$ 500 grams will not be more than 1 or 2 parts in 5000 and hence negligible, and this result can be attained without materially increasing the difficulty of the experimental work. Under these conditions the function can be treated as one of two variables and we obtain as allowable deviations in the temperature differences,

$$\frac{\delta(t_s - t_2)}{t_s - t_2} = \frac{\delta(t_2 - t_1)}{t_2 - t_1} = \frac{0.005}{\sqrt{2}} = 0.0035,$$

from which if $t_s - t_2 = 80°$ $\qquad \delta(t_s - t_2) = 0.29°$

and $t_2 - t_1 = 5°$ $\qquad \delta(t_2 - t_1) = 0.16°$.

If the allowable deviation in the difference of two quantities $t_2 - t_1$ is δ, then the allowable deviation in each of the component quantities is $\frac{\delta}{\sqrt{2}}$. Hence the allowable deviation in each reading t_1 and t_2 of the calorimeter thermometer will be $\frac{0.016°}{\sqrt{2}} = 0.011°$C. and in the readings t_s and t_2, $\frac{0.29}{\sqrt{2}} = 0.20°$ C.

Problem.— From a consideration of your data on the substance investigated and the above precision discussion, calculate how precise the following quantities should be determined, in order to ensure an accuracy of one per cent in the value of the specific heat:

a, the rise of temperature in the calorimeter;
b, the fall of temperature of the substance;
c, the weight of the substance;
d, the weight of water in the calorimeter.

LATENT HEAT OF VAPORIZATION

CONDENSATION METHOD

Object.— The object of this experiment is to determine the latent heat of vaporization of a liquid by the Condensation Method. The experiment affords an excellent example of a typical calorimetric measurement.

Discussion.—When a substance passes at constant temperature and pressure from one physical state to another, *e.g.*, from solid to liquid, or liquid to vapor, etc., a certain quantity of heat is absorbed or evolved, the amount of which corresponding to the transformation of one gram of the substance in question is known as the latent heat of the reaction. Thus if the reaction considered is the change of one gram of water from the liquid to the vapor state, the heat required to affect this change under the above conditions is called the latent heat of vaporization. It is the negative of the latent heat of condensation, *i.e.*, the heat evolved when one gram of saturated vapor condenses to one gram of liquid at constant pressure and temperature. The latent heat of any reaction consists in general of two parts, one corresponding to the energy involved in the molecular changes accompanying a change of physical state, and the other corresponding to the external work which is done by, or on, the system as a result of its change of volume against a constant pressure. The former may be spoken of as the change in the internal energy of the substance and will be denoted by e; the latter, the external work, is given by the expression $p\ (v_2 - v_1)$ where v_2 and v_1 are the specific volumes of the substance in the initial and final states, respectively. The latent heat may therefore be written

$$r = e + p\ (v_2 - v_1).$$

Of the terms in this fundamental expression, only r can be measured directly; e itself is not capable of direct measurement; $p\ (v_2 - v_1)$ can be at once computed when the pressure p and the specific volumes v_2 and v_1 are known. Thus in the case of the vaporization of water at 100° C., and under atmospheric pressure, 760 mm., $v_1 = 1.043$ $v_2 = 1661$ *cc.*, and the value of the external work in calories is 40.2. The total number of calories r required to vaporize one gram of water under these conditions is 536.5, hence of the total energy required to vaporize the water, 92.5 per cent is expended in doing internal work and 7.5 per cent in overcoming the atmospheric pressure during the expansion.

The latent heat of any reaction diminishes with rising temperature, being zero at the critical temperature for the particular change of state considered. From Regnault's data, Clausius has deduced the following empirical formula for the latent heat of vaporization of water between the temperatures 0° C. and 200° C.,

$$r = 607 - 0.708t.$$

If the latent heat of vaporization of a substance is multiplied by its molecular weight m, the resulting product $mr = \lambda$ is called the molecular heat of vaporization. Important stochiometrical relations have been found to hold for the values of this quantity for different liquids. Thus it is found that $\frac{\lambda}{T} =$ const. for a large class of analogous liquids, where T is the absolute boiling temperature. This relation is known as Trouton's law. The numerical value of the constant depends on whether the molecular weight of the liquid is equal to or greater or less than that of its vapor.

The most important methods for the determination of the latent heat of liquids may be classified as follows:

First.—The *Condensation Method* in which the quantity of heat given out by the condensation of a measured weight of vapor is determined.

Second.—The *Vaporization Method* in which the amount of heat absorbed by the evaporation of a measured weight of liquid is determined.

Third.—The *Electrical Method* in which the quantity of liquid vaporized at its boiling point by heat developed electrically in a heating coil placed within the liquid is measured.

Apparatus.—The apparatus used in this experiment is shown in figure 13. The water is boiled in the vessel *A* which is closed by a cover setting into a water seal *B*, thus preventing the escape of the steam at the top. The steam passes through a small tube *D* sealed tangentially into the cylinder *E*, a section of which is shown in figure 14. It circulates with considerable velocity around the inside of *E*, leaving it by means of the central tube from which it passes on to the calorimeter. The object of this device is to deprive the steam of any moisture which it may carry mechanically. This is deposited on the inner wall of the little cylinder and drops back into the water through small holes in the bottom. Unless the steam is "dry" an accurate determination of its latent heat is impossible

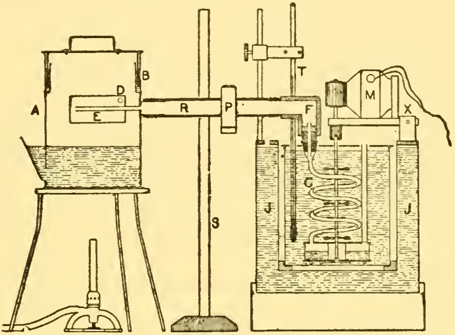

Fig. 13.

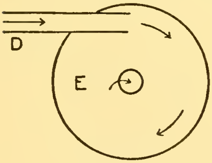

Fig. 14.

The calorimeter is of the closed type, of nickel-plated copper, and has a capacity of from one and one-half to two liters. It is shown in position for a run, resting on corks inside a water jacket *J*, which protects it from outside temperature changes. In the centre of the calorimeter is placed the condensing coil *C* which should be nearly completely immersed in water. In the base of the condenser is a bearing for the stirrer which is mechanically driven by means of a small electric motor so pivoted at *X* that it can be swung back out of the way, or connected with the stirrer, as desired.

The condenser is connected with the boiler by means of large heavy rubber tubing *R* and a brass elbow *F* lagged with felt to prevent condensation of the steam. The brass tube passing through the stoppers projects well up into *F* so as to form a trap for collecting any water which may become condensed before the steam reaches the condenser. The steam should enter the condenser well below the surface of the water, otherwise the water around the neck of the coil is likely to become unduly heated, thus causing excessive evaporation and consequent error in the temperature of the water in the calorimeter.

P is a clamp for closing the rubber connecting tube, and *S* is a screen for protecting the calorimeter from the heat of the burner and boiler.

Procedure.— *Preliminary.* See that all parts of the apparatus, water jacket, calorimeter, coil, stirrer and cover have the same number. Rinse out the boiler and fill it with about two inches of clean tap water. Fill the water seal, place the cover on the apparatus and allow the water to boil some time so that the steam may warm up the connecting tubes *R* and *F* before the experiment proper. In the mean time pour out any water which may be in the condenser and weigh the condenser to centigrams. The exact weight of the coil when perfectly dry is given with the apparatus and the difference between this weight and the weight found is to be taken as so much additional water in the calorimeter. Weigh the calorimeter, cover, and stirrer together, as they are all of the same material, to the nearest half gram. Next place the coil and stirrer in the calorimeter, fill the calorimeter to within a quarter of an inch of the top with clean tap water, put on the cover, and weigh all together to the nearest half gram. The weight of water can then be found by difference.

Next take the temperature of the water jacket, which should be nearly that of the room in which it stands continually. In the morning it will often be found below this temperature owing to the fall in temperature during the night. In this case it should be warmed by flashing a burner on the outside of the jacket, stirring the water within in the mean time by blowing into it through a glass tube. In this experiment the rise of temperature is at a uniform rate during the operation and hence the Rumford Method of correcting for radiation is applicable. As the temperature is to be allowed to rise not over 10° or 12° C., the initial temperature of the water in the calorimeter should be adjusted to about 5° or 6° below the temperature of the jacket. Under no circumstances should the temperature of the calorimeter be so low that moisture precipitates upon it from the atmosphere. The outside of the calorimeter and inside of the jacket should always be thoroughly dry. When the temperature is adjusted put the apparatus together as in the figure. Before connecting the coil with the boiler, the cover should of course be removed from the latter and the clamp *P* closed so that no steam can enter the condenser until the operation begins. Before connecting *F* to the coil it is advisable to first remove the stopper so as to release any water collected in the trap, otherwise it is likely to fill up during the operation and run over into the condenser. See that there is plenty of water in the boiler, and in the water seal before beginning the experiment proper, so that no further attention need be paid to these during or before the operation of passing steam into the coil. The water should be allowed to boil continually. The stirrer should be set in action several minutes before beginning to record temperatures.

Prepare a sheet for recording data in two columns, one for times (half minute intervals—which should be written down beforehand), and one for temperatures.

Experiment Proper.—The temperature readings are to be taken *without interruption* every half minute throughout the whole experiment.

First.—Take preliminary half-minute temperature readings of the calorimeter for six to eight minutes.

Second.—Open P, put on the cover of the boiler and note the instant when the temperature begins to rise suddenly. Continue reading the temperature of the calorimeter every half minute until the temperature has risen 10° or 12° C.

Third.—Take off the cover, close P and continue taking temperatures for six to eight minutes longer.

Fourth.—Remove the coil, dry the outside very carefully, and weigh as soon as possible to centigrams.

Fifth.—Record the reading of the barometer and its attached thermometer.

Computation.— Make a plot of the time and temperature observations and determine the corrected final temperature t_2 of the calorimeter by special method I or II (pp. 52–54); find from proper tables, the temperature t_s of the steam under the reduced barometric pressure at the time of the experiment.

Let r = latent heat;
w_c = weight of the condenser dry;
w = weight of water condensed in the coil;
w_o = weight of water in calorimeter;
$w_1 + w_2 + w_3$ = weight of calorimeter, cover and stirrer;
t_1 = corrected initial temperature of calorimeter;
t_2 = corrected final temperature of calorimeter;
t_s = corrected temperature of steam;
s_c, s_1, s_2, s_3 = specific heat of the condenser and parts of calorimeter, respectively. These may all be assumed equal to 0.095.

The heat given out by the steam in condensing to water at t_s° plus the heat given out by this water in cooling to t_2° is equal to the quantity of heat corresponding to the rise of temperature of the calorimeter and its contents, that is

$$rw + w(t_s - t_2) = (w_o + k)(t_2 - t_1),$$

from which $r = \dfrac{(w_o + k)(t_2 - t_1) - w(t_s - t_2)}{w}$

where k is the water equivalent of the calorimeter, cover, stirrer, and condenser, *i.e.*,

$$\begin{aligned} k &= w_c s_c + w_1 s_1 + w_2 s_2 + w_3 s_3 \\ &= 0.095\,(w_c + w_1 + w_2 + w_3). \end{aligned}$$

Problem.—From a consideration of the magnitude of the quantities measured in this experiment, calculate approximately how precise the following quantities should be determined in order to attain a precision of one-half of one per cent in the final value of the latent heat:

a. Rise of temperature in calorimeter;
b. Fall of temperature of condensed steam;
c. Weight of condensed water;
d. Value of total water equivalent $(w_o + k)$ of the calorimeter and contents.

MECHANICAL EQUIVALENT OF HEAT — I

Object.—The object of this experiment is to determine the value of the mechanical equivalent of heat by the electrical method. The experiment furnishes further practice in calorimetry and in the measurement of quantities of energy. A study of the electrical apparatus employed is also very instructive as illustrating the transformation of high potential direct currents to direct currents of low potential.

Discussion.—The mechanical equivalent of heat is the value of one calorie expressed in ergs, that is, the factor for reducing heat energy expressed in calories to mechanical energy expressed in ergs. Until recently all exact determinations of this quantity, Joule's, Rowland's, and others, consisted essentially in measuring the quantity of mechanical energy transformed by some device directly into heat energy, the amount of which was measured calorimetrically. If a ergs are found to produce b calories, by the principle of the conservation of energy, these two quantities of energy must be equal, *i.e.*, a ergs $= b$ calories, and the factor $J = \frac{a}{b}$ is therefore the desired factor for reducing calories to ergs.

The same result may be equally well obtained if some other form of energy, *e.g.*, electrical energy be transformed into heat, provided, first, that the relation of the units in which it is measured to the erg is exactly known, and second, that electrical energy can be measured with the same degree of accuracy as mechanical energy. With the refined methods and instruments of electrical measurement at our disposal, this is now possible, as recent determinations of the mechanical equivalent by wholly electrical means have shown.[1]

The determination of this fundamental physical constant with a high degree of accuracy requires elaborate apparatus and very careful manipulation. An accuracy of half of one per cent may be obtained, however, without much difficulty.

The method consists in measuring the electrical energy expended in heating a coil of wire placed in a calorimeter, on the one hand, and the heat energy thereby resulting, on the other. As the heating effect of a current is given by the expression $Q = IEt$ or $Q = I^2Rt$ the electrical input may be obtained by measurements either of current, voltage, and time, or current, resistance, and time. The former method is the more convenient and direct, as by it we measure the two factors which primarily constitute electrical energy, namely, potential and quantity of electricity (current × time). Moreover both of the quantities I and E can be determined with an accuracy of 0.1 or 0.2 per cent, by direct-reading instruments. The measurement of the heat energy is made in the usual way, all precautions necessary to accurate calorimetric measurements being observed. A precision of two or three tenths of a per cent can be obtained without serious difficulty.

Apparatus.—The apparatus consists of a closed calorimeter resting on cork supports inside a water jacket. The heating coil of flattened manganine wire fits concentrically inside the calorimeter. Manganine is employed on account of its very low temperature coefficient, and the wire is flattened so as to give a greater radiating surface. Inside the coil and supported by the frame is the stirrer, carrying two sets of vanes, which, when rapidly rotated by a small hot-air engine, keeps the water in violent agitation. The brass terminals of the coil project through insulated holes in the cover, and can, after the calorimeter, coil and cover are in place, be easily connected by flexible leads to the source of current.

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[1] Griffith, "Determination of Mech. Equiv. of Heat," Phil. Trans., **184**, p. 361, 1893.

The current is furnished by a motor-generator running on the 220 volt circuit and delivering energy at 12 volts. A low voltage is desirable in this experiment to avoid electrolysis. By means of a heavy fly wheel and the device of "floating" a set of batteries across the terminals of the heating coil, fluctuations arising from variations in the impressed voltage can be almost wholly eliminated. *The generator is to be started by an instructor* who will explain further details. A storage battery of large capacity is also very well suited for furnishing electrical energy for this experiment.

The current is measured by a Weston ammeter which is connected in series with the heating coil. The potential at the terminals of the heating coil is measured by a Weston voltmeter connected by potential leads to these points. The circuit is made and broken by a knife switch.

Procedure.— Two students will be assigned together to this experiment. Data should be recorded on blanks made out before the exercise, similar to those in the sample notebook.

Preliminary.—Weigh the calorimeter, cover, and coil dry. The weights and specific heats of the individual parts of the heating coil are given with the apparatus. Place the coil in the calorimeter and fill the latter with cold tap water so that the coil and frame are completely immersed, and weigh. The temperature of the water should be at least three or four degrees below that of the water in the jacket.

Set up the apparatus for a run. Start the hot air engine and stirrer. After the latter has been in action some minutes note the temperature of the calorimeter, and also that of the inside of the outer water jacket; the latter should be at about room temperature, and several degrees above that of the water in the calorimeter. Since in this experiment the rate at which heat is put into the calorimeter is practically uniform from start to finish, the Rumford method of eliminating the radiation correction may be advantageously employed. The rise of temperature in the calorimeter should be about five degrees; the initial temperature of the calorimeter should therefore be brought up to within 2.5° of that of the surroundings. This can be done exactly by closing the circuit for a few minutes and allowing the current to heat up the water in the calorimeter by the proper amount. *The circuit should never under any circumstances be closed unless the heating coil is wholly immersed in water.* The apparatus and connections should be inspected by an instructor before the circuit is closed.

Experiment proper.—Having adjusted the initial temperature of the water in the calorimeter, all is ready for the run. One student A, is to open and close the circuit and read the voltmeter and ammeter; the other B, is to read the thermometer. Each should prepare blanks beforehand for recording his observations. Temperatures are to be taken every half minute, and voltmeter and ammeter readings alternately every fifteen seconds. A preliminary series of temperature readings are to be taken for five minutes before closing the circuit; then at a noted instant, preferably *on* the minute, the circuit is closed, and voltmeter and ammeter read instantly. When the temperature has risen nearly five degrees, B, reading temperatures, should notify A, who, at a noted instant, opens the switch. The switch should be opened and closed as nearly at the noted times as possible, so as to reduce the error in the time measurement to a minimum. The temperature of the calorimeter should be continuously observed five minutes longer to provide all necessary data for correcting for radiation and mechanical heating due to stirring, if these be not wholly eliminated by the above-described method of procedure. This completes one set of data.

The water in the calorimeter should then be cooled to a temperature below that of its surroundings and a duplicate run made in exactly the same manner, A and B changing places.

Computation.— The calorimetric data are to be plotted and the corrected rise of temperature determined as described under special case, I or II, pp. 52–54. The computation of the electrical energy is to be made as follows: Since all ammeter and voltmeter readings are taken at equal time intervals, the mean voltage and current multiplied by the total time may be taken without error for the sum of the products of each pair of readings multiplied by the corresponding time interval. The mean value of the current and voltage must be corrected for instrumental errors. The calibration corrections of the instruments used will be found with the instruments. The "zero error" should be observed by the student. Without careful calibration, the instruments cannot be assumed correct to 0.1–0.2 per cent.

The product, current (in amperes) × potential (in volts) × time (in seconds) gives the total electrical energy expended in heating the calorimeter, expressed in joules. Equating this quantity to the number of calories actually measured, the value of one calorie expressed in the *practical* unit of electrical energy, the joule is readily obtained. To reduce this value to the absolute unit of energy, the erg, we must know the value of a joule in ergs. The system of electrical units is so *defined* that one joule $= 10^7$ ergs.

Each student is to compute one run independently, but the final results of both are to be handed in for comparison.

Problems.—How many joules are necessary to vaporize 100 grams of water at its boiling point under standard conditions of pressure? If this energy is supplied at the rate of 100 watts, what time will be necessary? If the resistance of the coil is 10 ohms, what current and voltage will be necessary? With a 110-volt circuit what must I and R be?

2. What must be the percentage precision of I, E, and t in order that the resultant deviation in Q is not greater than 0.5 per cent?

3. What are the allowable deviations in amperes, volts, and seconds corresponding to the precision prescribed in problem 2 if the value of I, E, and t are the mean values found in the experiment? Do you think this degree of precision was attained?

MECHANICAL EQUIVALENT OF HEAT — II

CONTINUOUS CALORIMETER

Object.— The object of this experiment is to give practice in the use of a *continuous calorimeter* for measuring heat energy. The method is illustrated by a determination of the mechanical equivalent of heat by the electrical method.

Discussion.— The principle of a *continuous calorimeter* is as follows: Heat is generated at a uniform rate within a calorimeter which is so constructed that the heat can escape only by conduction and radiation through a stream of water which circulates through the apparatus. The temperature of the incoming water must be below the temperature of the room (surroundings); its rate of circulation through the calorimeter is so regulated that its temperature when it leaves the apparatus has been raised just to the temperature of the room. Under these circumstances the outflowing water conducts away, at a constant rate, the heat generated within the calorimeter, and the amount of heat generated in a given time will be measured by the quantity of water flowing in that time, multiplied by the rise of temperature which the water has experienced in passing through the apparatus. A determination of the difference in temperature of the incoming and outgoing water when a state of equilibrium has been established in the calorimeter, and of the quantity of water flowing through in a given time, furnishes therefore all data necessary for computing the rate of heat development within the calorimeter.

A little consideration will show that, in order that a state of thermal equilibrium may be established within the apparatus the following conditions must be fulfilled:—

(*a*) The heat must be generated uniformly.

(*b*) The water must flow uniformly, *i.e.*, under a constant pressure.

(*c*) The temperature of the incoming water must be constant.

(*d*) The temperature of the surroundings must be constant.

Under these conditions it is to be particularly noted that the usual correction for radiation, so troublesome in most calorimetric experiments, may be entirely eliminated, for, if the outgoing water is caused to circulate within the outside wall of the calorimeter, and its temperature is maintained the same as the temperature of the surrounding air, there will be no exchange of heat by the apparatus either by radiation or conduction to or from the surroundings, no matter what may be the duration of the experiment.

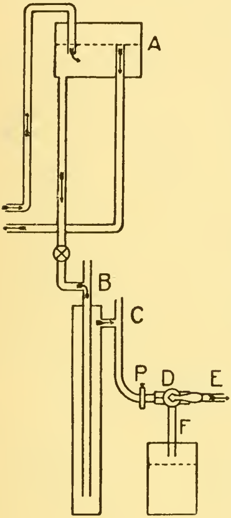

Fig. 15.

The apparatus described below is designed to meet the above requirements, and to give the calorimetric results reliable to at least 0.5 per cent.

Apparatus.— The general arrangement of the apparatus is shown in Fig. 15. The water is piped directly from the street mains, in order to insure a constant temperature below that of the laboratory, into an overflow tank *A*, placed about ten feet above the apparatus, which furnishes a supply at a constant pressure. The water from the tank flows down into the calorimeter at *B*, circulating up and down around an enclosed heating coil, as shown in Fig. 16, and finally leaves it at *C*. *D* is a three-way cock, by means of which the water can be run off into the waste pipe *E*, or collected through *F*. The temperature of the incoming water is taken by the thermometer inserted at *B*, and of the outgoing water at *C*.

The details of the calorimeter are shown in Fig. 16. It consists of a small inner brass tube around which, but insulated from it, are wound a number of turns of fine german-silver wire. In order to insulate the wire and also to facilitate the conduction of the heat generated in it to the water, the coil is completely immersed in oil.

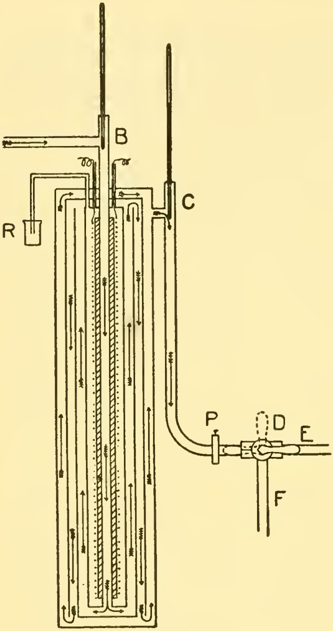

Fig. 16.

The incoming water passes down through the heating coil and then circulates up and down through the calorimeter as shown in the figure. A pinch-cock regulates the flow at *P*. A small reservoir *R*, connected to the oil-jacket, serves to receive the overflow due to the expansion of the oil when heated by the current in the coil.

The calorimeter is placed inside a metal jacket to protect it from draughts. Both calorimeter and jacket are nickel-plated and highly polished to diminish the radiation. A vessel for collecting the water and suitable scales for weighing it are provided.

The electrical measuring instruments required consist of a portable Weston ammeter and volt-meter, the former being connected in series with the heating coil, and the latter across its terminals. The coil is connected directly to the 110-volt mains. The instruments are calibrated and their corrections will be found on them.

Procedure.— This experiment is to be performed by two students working together. Data should be recorded similar to those in the sample note-book. All pipe and electrical connections should be explained by an instructor before the experiment is begun.

Preliminary.—Allow the water to flow through the apparatus with the pinch-cock *P*, which regulates the outflow of the water from the calorimeter, wide open so that a full stream runs through the calorimeter into the waste pipe *E*. Close the circuit containing the heating coil.

When the temperature of the inflowing water has become constant to 0°.2 for at least two minutes, begin regulating the outflow by the pinch-cock *P* so that its temperature approaches that of the room. While waiting for the outflowing water to come to a constant temperature, weigh the vessel for collecting the water and arrange the blanks for recording the observations.

When the outflowing water has come to a constant temperature, note whether it is above or below that of the surroundings, *i.e.*, the temperature of the air just inside the calorimeter jacket. If above, open the pinch-cock slightly to allow the water to flow more rapidly, and wait until the temperature again becomes constant. If below, close the pinch-cock slightly, and wait as before. A fraction of a revolution of the pinch-cock screw will be found to produce a considerable variation in the ultimate temperature of the outgoing water. When the stationary condition of temperature is reached, the temperature of the outgoing water should not vary more than 0°.1 in three minutes. Continue this operation (being sure to wait long enough after each adjustment for equilibrium to be established) until the temperature of the outgoing water is *within* 0°.2 *of that of the surroundings.* When this condition is reached, but not before, begin the experiment proper.

Experiment proper.—One student will give the signals and note the times of starting and stopping the run, and will record readings alternately of the voltmeter and ammeter every fifteen seconds for ten minutes. The other student will begin to collect and stop collecting the water at the signals given, weigh the water, and record the temperatures of the ingoing and outcoming water alternately at intervals of fifteen seconds during the run. The temperature of the surroundings should also be recorded at the beginning, middle, and end of the run.

A short time, half a minute or so, before beginning the run, turn the three-way cock so that the water will flow out through *F* into any convenient vessel. Then at the signal for starting the run, begin to collect the water for weighing by quickly placing an appropriate vessel of four litres or more capacity under *F*. This procedure for collecting the water has been found to introduce less initial error than turning the three-way cock from *E* to *F* at the starting signal. At once begin to read the temperatures of the incoming and outgoing water. At the signal for concluding the run (duration about ten minutes), stop collecting the water by pinching the end of the rubber tube with the fingers, and at once turn the three-way cock so that the water will again flow into the waste pipe *E*. Weigh the quantity of water collected.

This completes one set of data. The run should be repeated, the two observers changing places.

Computation.—The electrical input is to be computed from the mean ammeter and voltmeter readings corrected for zero and calibration errors, and the time. The corresponding value of the heat energy evolved is to be computed from the mean difference of temperature of the incoming and outgoing water and its weight. If the temperature readings are nearly constant, as they should be, the difference of the mean temperature of the incoming and outgoing water may be used in the computation. The thermometers used must be very carefully compared with each other. The corrections are given.

Each student is to compute one run independently, but the final results of both are to be handed in for comparison. The problems on page 71 should also be included unless previously handed in.

EXPANSION

The determination of the coefficients of expansion of liquids, solids, or gases with any high degree of precision requires skillful manipulation and attention to numerous details and corrections, and not infrequently the use of least squares in the reduction of the data. Special apparatus including comparators, and Abbé's Dilatometer (Fizeau's apparatus), is provided for this work, which is offered to students who are qualified to undertake it. The work is not required however in the introductory laboratory course.

APPENDIX

The data in the following abbreviated tables are taken from Landolt and Börnstein's "Physikalisch-chemische Tabellen."

TABLE I.

LINEAR COEFFICIENTS OF EXPANSION.

Aluminium	0.000023	Lead	0.000029
Brass	0.000019	Platinum	0.000009
Copper	0.000017	Platinum-iridium	0.000009
German Silver	0.000018	Silver	0.000019
Glass, ordinary	0.0000085	Tin	0.000023
Gold	0.000015	Vulcanite	0.00008
Iron	0.000012	Zinc	0.000029

Wood with the grain	0.000003 to 0 000010
Wood across the grain	0.000033 to 0.000061

TABLE II.

MEAN CUBICAL COEFFICIENTS OF EXPANSION BETWEEN 0° AND 100°.

Benzene	0.00138	Mercury	0.000181
Toluene	0.00121	Ether	0.00215
Turpentine	0.00105	Alcohol	0.00104
Chloroform	0.00140	Glycerine	0.000534

For *water*, $V_t = V_o(1 + at + bt^2 + ct^3)$.
where $a = -0.00006581$.
$b = +0.00000851$.
$c = -0.000000068$.
v_o = volume at 0° C.
v_t = volume at t° C.

TABLE III.

MEAN COEFFICIENTS OF EXPANSION OF GASES BETWEEN 0° AND 100°.

	Volume Constant.	Pressure Constant.
Air	0.003669	0.003671
Oxygen	0.003674	
Hydrogen	0.003668	0.003661
Nitrogen	0.003668	
Carbonic Oxide	0.003667	0.003669
Carbonic Acid	0.003706	0.003710
Nitrous Oxide	0.003676	0.003720

TABLE IV.

MELTING POINTS (Holborn & Day, 1901).

Cadmium	321.°7 ± 0.°1	(From 10 observations on two days.)
Lead	326.°9 ± 0.°2	" " " " " "
Zinc	419.9 ± 0.2	
Antimony	630.5 ± 0.3	
Aluminium	657.5 ± 0.5	(In a graphite crucible, a lower value.
Silver	961.5 ± 0.9	(Pure; *i.e.*, in a reducing atmosphere.
Silver	955	(In air; point ill defined.)
Gold	1064	
Copper	1065.	(In air.)
Copper	1084.	(Pure.)

TABLE V.

BOILING POINTS OF LIQUIDS AT 760 mm. PRESSURE.

Water	100 °C.	Benzene	80.°0
Mercury	357.°3	Glycerine	290.°0
Sulphur	444.°0	Alcohol	78.°2
Naphthaline	218.°0	Turpentine	159.°0
Toluene	110.°0	Chloroform	61.°2

TABLE VI.

LATENT HEATS OF VAPORIZATION EXPRESSED IN 0° CALORIES.

	Temperature of Vaporization.	Latent Heat.
Mercury	350	62.00
Alcohol	78	206.4
Ether	34.9	89.96
Toluene	110.8	83.55
Turpentine	159.3	74.04
Ammonia	7.8	294.2
Benzene	80.1	92.91
Water	100	536.2 (15° to 16°)

Clausius's formula for the variation of the latent heat of vaporization of water with the temperature is, $r = 607 - .708t$ when t is the temperature of vaporization in degrees centigrade.

TABLE VII.

SPECIFIC HEATS OF VARIOUS LIQUIDS AND SOLIDS.

Alcohol at 17°	0.58	Glass between 15-100°	0.20
Aniline " "	0.49	Gold " "	0.032
Benzene " "	0.36	Iron " "	0.113
Mercury " "	0.034	Lead " "	0.032
Toluene " "	0.40	Nickel " "	0.11
Turpentine at 17°	0.43	Platinum " "	0.033
Aluminium between 15-100°	0.218	Quartz " "	0.191
Brass " "	0.093	Silver " "	0.057
Fluorspar " "	0.208	Tin " "	0.056
Copper " "	0.093	Zinc " "	0.094

TABLE VIII.

TEMPERATURE, PRESSURE, AND SPECIFIC VOLUME OF SATURATED VAPOR.[1]

Temperature in Degrees Centigrade.	Pressure in mm. of Mercury.	Specific Volume in c.cm.	Temperature in Degrees Centigrade.	Pressure in mm. of Mercury.	Specific Volume in c.cm.	Temperature in Degrees Centigrade.	Pressure in mm. of Mercury.	Specific Volume in c.cm.
0	4.602	211500	38	49.308	21860	76	300.83	3965
1	4.941	197700	39	52.05	20770	77	313.59	3813
2	5.303	184600	40	54.91	19740	78	326.80	3668
3	5.689	172400	41	57.92	18760	79	340.48	3529
4	6.100	161200	42	61.06	17840	80	354.63	3397
5	6.536	150800	43	64.35	16980	81	369.27	3270
6	7.001	141200	44	67.80	16160	82	384.41	3149
7	7.494	132200	45	71.40	15390	83	400.08	3033
8	8.019	123900	46	75.16	14660	84	416.27	2922
9	8.576	116200	47	79.10	13970	85	433.01	2815
10	9.167	109000	48	83.21	13310	86	450.31	2714
11	9.795	102300	49	87.51	12690	87	468.18	2616
12	10.460	96090	50	91.98	12110	88	486.64	2523
13	11.164	90190	51	96.65	11560	89	505.71	2433
14	11.911	84760	52	101.54	11030	90	525.40	2347
15	12.702	79690	53	106.64	10530	91	545.72	2265
16	13.539	74970	54	111.95	10060	92	566.70	2186
17	14.423	70560	55	117.49	9610	93	588.34	2110
18	15.360	66440	56	123.25	9185	94	610.67	2038
19	16.349	62580	57	129.26	8782	95	633.70	1968
20	17.395	58980	58	135.51	8399	96	657.45	1901
21	18.498	55610	59	142.02	8036	97	681.93	1836
22	19.663	52460	60	148.80	7687	98	707.17	1774
23	20.892	49510	61	155.85	7362	99	733.19	1715
24	22.188	46740	62	163.18	7051	100	760.00	1661
25	23.554	44150	63	170.80	6754	101	787.5	1609
26	24.994	41720	64	178.72	6470	102	815.8	1556
27	26.510	39450	65	186.95	6201	103	845.0	1505
28	28.107	37310	66	195.50	5947	104	875.1	1456
29	29.786	35300	67	204.38	5705	105	906.0	1409
30	31.553	33420	68	213.60	5472	106	937.9	1365
31	33.411	31650	69	223.17	5250	107	970.7	1320
32	35.364	29980	70	233.09	5040	108	1004.4	1278
33	37.416	28420	71	243.39	4839	109	1039.1	1248
34	39.571	26940	72	254.07	4648	110	1074.7	1209
35	41.833	25560	73	265.14	4465	111	1111.4	1162
36	44.207	24250	74	276.62	4291	112	1149.1	1126
37	46.697	23020	75	288.51	4124	113	1187.9	1091

[1] These values are taken from Peabody's Steam-Tables.

MECHANICAL EQUIVALENT OF HEAT.

1 kilogram-calorie (1 kilogram water raised 1° C. at 15° C.) = 427.3 kilogrammeters (at sea-level, latitude 45°, $g = 980.6$ c. g. s.).

1 British thermal unit (1 pound water raised 1° F. at 59° F.) = 778.8 foot pounds at sea-level, latitude 45°.

1 gram-calorie (1 gram of water raised 1° C. at 15° C.) = 4.190×10^7 ergs.

1 Joule = 10^7 ergs.

= 0.2387 gram-calorie.

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Goodwin, Harry Manley, 1870-
Physical laboratory experiments : mechanics, optics and heat / by H. M. Goodwin. 6th ed. Boston : G. H. Ellis Co., Printers, 1917.
119, ix, 46, 81 p. : ill. ; 23 cm.

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3 9358 00013169 5